COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **1**, No. 6, pp. 974-983

Commun. Comput. Phys.
December 2006

On the Naturally Induced Sources for Obstacle Scattering

Perrin S. Meyer[1], Marcos Capistran[2] and Yu Chen[3,*]

[1] *Meyer Sound, 2832 San Pablo Avenue, Berkeley, CA 94702-2204, USA.*
[2] *Universidad Autonoma del Estado de Morelos, Av. Universidad No. 1001, Col. Chamilpa, 62210 Cuernavaca, Morelos, Mexico.*
[3] *Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012-1110, USA.*

Received 14 November; Accepted (in revised version) 8 May 2006

Abstract. We introduce the equivalent sources for the Helmholtz equation and establish their connections to the naturally induced sources for the sound-soft, sound-hard, and impedance obstacles for the inverse scattering problems of the Helmholtz equation. As two applications, we employ the naturally induced sources to improve the boundary integral equation formulations for the obstacle scattering problems, and develop a unified, straightforward approach to establishing boundary conditions governing the domain derivatives of scattered waves for the soft, hard, and impedance obstacles.

Key words: Naturally induced sources; obstacle scattering; domain derivatives.

1 Introduction

The subject of this paper is on the forward and inverse obstacle scattering problems for the Helmholtz equation. We will introduce the notion of naturally induced sources in the scattering by an obstacle, and use it to reformulate the standard boundary integral equations for the forward scattering problems. We also use it to establish a unified approach to the domain derivatives for the inverse obstacle scattering problems.

An equivalent source for a time harmonic wave u_0 in a domain D is made of monopoles, dipoles, or their combination on the boundary which reproduces the wave in the domain. The problems of determining the equivalent sources given u_0 is referred to as the interior (scattering) problems. There are three standard interior problems for the monopoles, dipoles, and their linear combination.

*Correspondence to: Yu Chen, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012-1110, USA. Email: `yuchen@cims.nyu.edu`

If the domain is the support of an obstacle, of sound-soft or sound-hard or impedance type, the scattered wave can be expressed as the potential of the single or double or combined layer, respectively. These monopole, dipole, and combined sources are referred to as the naturally induced sources for the soft, hard, and impedance obstacles, respectively. Thus, for instance, double-layer potential for the exterior Neumann problem of the Helmholtz equation employs the naturally induced (dipole) sources, whereas combined potential for the exterior Dirichlet problem does not.

`The interior-exterior connection.` We will establish connections between these interior and exterior problems by identifying the naturally induced sources for the soft, hard, impedance obstacles with the equivalent sources of monopole, dipole, combined types, respectively. There are two applications of the interior-exterior connection.

`Reformulate scattering problems.` It is typical in the scattering problems that u_0 arises from known sources outside D, so both u_0 and $\partial_n u_0$ are available on the boundary. We will make use of this flexibility and employ the equivalent sources to rewrite boundary integral equations for the exterior problems. The reformulated problems are more convenient to solve, or their solution - once obtained - easier to process, than the standard approaches.

`Domain derivatives.` The calculation of domain derivative, or more generally of the Frechet derivative of the scattered wave with repect to perturbation to the bundary of the obstacle, is an essential step for the inverse obstacle problem. With the help of the equivalent sources, we will present a unified, straightforward approach to establishing boundary conditions for the domain derivatives of the scattered waves off the soft, hard, and impedance obstacles. The domain derivatives for these three scattering problems, and for the transmission problem, have already been characterized by a number of authors, initiated by the work of Kirsch [3]; see Haddar and Kress [4] for a quite complete description of the existing work.

`Organization of paper.` Section 2 provides preliminary tools. Section 3 introduces the equivalent sources and naturally induced sources, and makes the interior-exterior connection. In Section 4 we reformulate the obstacle scattering problems. Section 5 presents a straightforward, unified approach to calculating the boundary values of the domain derivatives for the soft, hard, and impedance obstacles.

2 Analytic machinery

Let D be a domain and $k > 0$ be the wave number. Throughout the paper, we work with smooth boundary ∂D; we also assume that the incident wave u_0 is generated by sources away from the boundary, so that the classical theory for layer potentials holds and that the scattering solutions are smooth. This section summarizes basic facts for layer potentials and perturbational properties of the boundary ∂D.