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A Moving Mesh Method for the Euler Flow Calculations Using a Directional Monitor Function

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Abstract. This paper is concerned with the adaptive grid method for computations of the Euler equations in fluid dynamics. The new feature of the present moving mesh algorithm is the use of a dimensional-splitting type monitor function, which is to increase grid concentration in regions containing shock waves and contact discontinuities or their interactions. Several two–dimensional flow problems are computed to demonstrate the effectiveness of the present adaptive grid algorithm.

Key words: Euler equations; adaptive grid method; finite volume method; monitor function.

1 Introduction

In fluid dynamics, the physical solutions usually develop dynamically singular or nearly singular solutions in fairly localized regions, such as shock waves, detonation waves, contact discontinuities, and boundary layers, etc. To resolve these large solution variations, the numerical simulations may require extremely fine meshes on a small portion of the physical domain. It becomes very expensive for computations of multi–dimensional problems if a uniform mesh is used. It is therefore very necessary to develop an effective, robust, multi-dimensional adaptive grid methods. Successful implementation of the adaptive strategy can increase the accuracy of the numerical approximations and also decrease the computational costs.

Moving mesh methods are a class of adaptive grid methods, which have important applications in fluid dynamics. They include the variational grid methods or adaptive grid generation methods, the traditional Lagrange methods, and their variations such as

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arbitrary Lagrangian–Eulerian methods, the free–Lagrange methods, and the unified coordinate system methods. In the past several decades, there has been important progress in moving mesh methods for partial differential equations, including the variational approach of Winslow [33], Brackbill et al. [3, 4], Dvinsky [9], and Li et al. [16, 17]; moving finite element methods of Miller et al. [21], and Davis and Flaherty [8]; and moving mesh PDEs of Cao et al. [6], Li and Petzold [18], and Ceniceros and Hou [7]. We refer the readers to a recent paper [31] for a detailed review. Some recent work on the Lagrange methods can be found in [2, 12, 13, 22] and references therein.

The monitor function is one of the most important issues in the adaptive moving mesh algorithms. The appropriate choice of the monitor will generate grids with good quality in terms of smoothness, skewness, and aspect ratio. Cao et al. [5] gave a general strategy of choosing the monitor function. Some general forms of the monitor function can also be found in [19]. The conventional monitor functions usually depend on the magnitude of the gradient of the solutions, for example, $\sqrt{1+\alpha|\rho|^2+\beta|\nabla\rho|^2}$. However, the gradient-monitor is not always successful in increasing grid concentration in regions containing shock waves and contact discontinuities or their interactions. Our numerical results will demonstrate this phenomenon. To overcome this drawback, a dimensional–splitting type monitor function will be considered. The moving mesh equations used in this paper is multi–dimensional and more robust than the multidimensional equidistributed methods, see [19].

In this paper, we are interested in developing moving mesh methods based on a variational approach for the hyperbolic conservation laws including the Euler equations of gas dynamics. Harten and Hyman [11] began the earliest study of the self-adaptive moving mesh methods to improve resolution of shock and contact discontinuity. They applied the Godunov method on a non-uniform mesh where the grids move along the characteristic direction. After their work, many other moving mesh methods in this direction have been proposed in the literature based on combining the variational grid methods with high resolution shock capturing methods. They include those of Azarenok et al. [1], Fazio and LeVeque [10], Liu et al. [20], Saleri and Steinberg [23], Stockie et al. [26], and Zegeling [35]. However, it is noticed that many existing moving mesh methods for hyperbolic problems are designed for one space dimension. In 1D, it is generally possible to compute on a very fine grid and so the need for moving mesh methods may not be clear. Multidimensional moving mesh methods are often difficult to be used in fluid dynamics problems since the grid will typically suffer large distortions and possible tangling. It is therefore useful to design a simple and robust moving mesh algorithm for computational problems in fluid dynamics.

Recently, an adaptive moving mesh method for multidimensional hyperbolic conservation laws was proposed by Tang and Tang [28]. The moving mesh algorithm includes two parts: PDE evolution and mesh redistribution. The PDE evolution may be any appropriate high resolution finite volume scheme. The mesh redistribution is an iterative procedure. In each time iteration, grid points are first redistributed by a variational principle, and then the numerical solutions are updated on the resulting new meshes by a