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A Generalised Lattice Boltzmann Equation on Unstructured Grids

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Abstract. This paper presents a new finite-volume discretization of a generalised Lattice Boltzmann equation (LBE) on unstructured grids. This equation is the continuum LBE, with the addition of a second order time derivative term (memory), and is derived from a second-order differential form of the semi-discrete Boltzmann equation in its implicit form. The new scheme, named unstructured lattice Boltzmann equation with memory (ULBEM), can be advanced in time with a larger time-step than the previous unstructured LB formulations, and a theoretical demonstration of the improved stability is provided. Taylor vortex simulations show that the viscosity is the same as with standard ULBE and demonstrates that the new scheme improves both stability and accuracy. Model validation is also demonstrated by simulating backward-facing step flow at low and moderate Reynolds numbers, as well as by comparing the reattachment length of the recirculating eddy behind the step against experimental and numerical data available in literature.

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Key words: Lattice Boltzmann equation, finite-volumes, unstructured grids, memory term.

1 Introduction

In the last decade, the lattice Boltzmann method (LBM) has become an established numerical approach in computational fluid dynamics. Many models and extensions have been formulated that cover a wide range of complex fluids and flows [1, 2]. The LBM, that originally evolved from lattice gas models, is based on a minimal kinetic Boltzmann

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equation in which representative particles ('parcels of fluids') evolve on a regular cartesian grid according to simple streaming and collide rules, designed in such a way as to preserve the basic symmetries (conservation laws) of fluid dynamics. This method possesses some advantages over conventional CFD methods, such as the simplicity of the stream-and-collide dynamics that makes LB very efficient from the computational point of view, its amenability to parallel computing, its ease in handling complex flows and the physical implementation of complex boundary conditions. However, the essential restriction of the standard LBE to the lattice uniformity, which makes it macroscopically similar to a uniform Cartesian-grid solver, represents a severe limitation for many practical engineering problems. Therefore, in the recent years, much research has been directed to the goal of enhancing the geometrical flexibility of the LB method [3,4,15,27].

Considering that for many practical problems an irregular grid or a meshless structure is always preferable due to the fact that curved boundaries can be described more accurately, and that computational resources can be used more efficiently, our recent effort is to extend the LBM order of accuracy and flexibility so that its spatial resolution requirements for various flow situations may be reduced and may be adapted to more general meshes. Indeed, starting from the earliest finite-volume formulations more than a decade ago [3], today many options are available to deal with realistically complex geometries [5–8]. A particularly interesting development is represented by finite-volume formulations on fully unstructured grids [9–11] which were recently extended to 3D grids [12]. The Unstructured Lattice Boltzmann schemes (ULBE for short) integrates the differential form of the Lattice Boltzmann equation (LBE) using a cell-vertex finite-volume technique in which the unknown fields are placed at the nodes of the mesh and evolve based on the fluxes crossing the surfaces of the corresponding control volumes. These finite-volume formulations are best viewed as a coarse-grained version of the original LB dynamics, in which geometrical flexibility is achieved at the level of the coarse-graining elements, whose triangular (in two dimensions) or tetrahedral (in three dimensions) shapes can accommodate the most complex geometries. Even if the ULBE method is less efficient than the standard LBE in updating the single node, computational savings are expected whenever the number of grid nodes can be reduced by, say, an order of magnitude as compared to cartesian grids. Whether or not such a reduction can be achieved depends of course on the geometrical complexity of the problem at hand, but it is reasonable to expect that for highly complex geometries ULBE should indeed gain a significant potential.

Nevertheless, the standard ULBE scheme suffers of the significant limitation $\Delta t < 2\tau$ which implies an adverse scaling of the time-step with the inverse Reynolds number [11,12]. In order to overcome the above mentioned stability limit a new scheme, called ULBE with memory (ULBEM for short), has been developed [23] by introducing a 2^{nd} order term on the continuous lattice Boltzmann equation to be numerically solved through a finite volume approach. This generalised formulation of the continuous lattice Boltzmann equation achieves better numerical stability and higher numerical accuracy, while maintaining almost the same computational costs of the standard ULBE scheme. In this