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An Optimization-Based Rezoning for ALE Methods†

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Abstract. Based on the theory of optimization, we use edges and angles of cells to represent the geometric quality of computational grids, employ the local gradients of the flow variables to describe the variation of flow field, and construct a multi-objective programming model. The solution of this optimization problem gives appropriate balance between the geometric quality and adaptation of grids. By solving the optimization problem, we propose a new grid rezoning method, which not only keeps good geometric quality of grids, but also can track rapid changes in the flow field. In particular, it performs well for some complex concave domains with corners. We also incorporate the rezoning method into an Arbitrary Lagrangian-Eulerian (ALE) method which is widely used in the simulation of high-speed multi-material flows. The proposed rezoning and ALE methods of this paper are tested by a number of numerical examples with complex concave domains and compared with some other rezoning methods. The numerical results validate the robustness of the proposed methods.

AMS subject classifications: 76M12, 76M10, 76N15, 65K10, 90C29

Key words: Grid rezoning, multi-objective programming models, ALE methods.

1 Introduction

Multi-material flows, where a moving interface exists between two immiscible fluids, can be found in a variety of scientific and engineering problems. Development of numerical accurate and computationally efficient algorithms for multi-material flow simulations remains one of the challenging topics in computational fluid dynamics. Traditionally, numerical methods for multi-material computations have fallen into two classes: Eulerian methods and Lagrangian methods. Eulerian methods hold the mesh of cells fixed and a fluid flows from one cell to another through cell edges via advection. Eulerian methods are robust, capable of running under severe flow conditions (such as under large flow

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deformation), but may result in badly smeared material interfaces due to numerical diffusion. For Lagrangian methods, cells flow with the fluid and no fluid moves across cell edges. Material interfaces remain intact as they travel with cells. Lagrangian methods are capable of producing sharp interfaces, but may result in mesh contortion and tangling, causing inaccuracy and even breakdown of computation. Over the last decades, another method has been developed that smoothly spans Eulerian and Lagrangian methods offering the benefits of both: the Arbitrary Lagrangian-Eulerian (ALE) method proposed by Hirt, Amsden and Cook (see [25,33]). In the ALE method, the solution algorithm can vary from pure Eulerian to pure Lagrangian through dynamic rezoning and remapping, such that a smooth mesh topology can be maintained, increasing thus accuracy and robustness of the numerical algorithm. A general review of the ALE method can be found in the paper by Benson [9]. There are further developments and applications of this approach, see, e.g., [4,19,23,28,32,34,37].

Generally, an ALE method consists of three phases: The explicit Lagrangian phase, the rezoning phase (mesh movement) and the remapping phase. One of the key factors to a successful ALE method is a robust rezoning algorithm in the rezoning phase that does not require user intervention. In the early development of the ALE methods, the rezoning phase was often carried out by employing a process of grid generation, for which only the geometric quality of the grid was taken into account [1,17,20]. However, the geometric quality of a grid is not the only factor that will affect simulation results. From the numerical simulation point of view, it has become a common sense that a good rezoned grid should in general satisfy the following four requirements:

(i) A rezoned grid should remain convex. A lack of control of grid skewness may result in a major deficiency for some algorithms (see, e.g., [2]).

(ii) A rezoned grid should maintain the smoothness, orthogonality and uniformity to increase the computational accuracy. The geometric quality of a grid affects the accuracy of the Lagrangian phase in the ALE methods. In a non-Descartian grid, the numerical error of the Lagrangian phase is not only induced by the truncation error of the used schemes and the grid size, but also depends on the smoothness, orthogonality and uniformity of the grid.

(iii) In the regions where the gradients of the flow variables are large, the distance between the rezoned grid and the old grid must be small in order to keep the remapping error small. Some rezoning methods require that the rezoned grid should be close to the Lagrangian grid, see, e.g., [22, 28, 42], and this idea works well in many cases. But, in some cases other criteria of rezoning are better, see, e.g., [31]. By numerical tests we have found that the distance between the old grid and the rezoned grid plays an important role in resolving local gradients of the flow variables in the remapping phase.

(iv) A rezoned grid should be adaptive to resolve local gradients of the flow variables. Recent progress in the development of r-adaptive methods (i.e., moving mesh methods) shows that higher accuracy can be achieved by appropriate moving cells to regions of rapid changes in the flow variables, see, e.g., [5,26,30,37,40].

In view of the above requirements and analysis, we see that a suitable grid movement