COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **3**, No. 4, pp. 759-793

Commun. Comput. Phys.
April 2008

REVIEW ARTICLE

Some Mathematical and Numerical Issues in Geophysical Fluid Dynamics and Climate Dynamics

Jianping Li[1] and Shouhong Wang[2,*]

[1] *State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics (LASG), Institute of Atmospheric Physics (IAP), Chinese Academy of Sciences (CAS), P.O. Box 9804, Beijing 100029, China.*
[2] *Department of Mathematics, Indiana University, Bloomington, IN 47405, USA.*

Received 11 January 2007; Accepted (in revised version) 10 November 2007

Communicated by Jie Shen

Available online 11 December 2007

Abstract. In this article, we address both recent advances and open questions in some mathematical and computational issues in geophysical fluid dynamics (GFD) and climate dynamics. The main focus is on 1) the primitive equations (PEs) models and their related mathematical and computational issues, 2) climate variability, predictability and successive bifurcation, and 3) a new dynamical systems theory and its applications to GFD and climate dynamics.

AMS subject classifications: 86A05, 86A10, 76D, 76E, 76U05, 37L, 37M20, 35Q, 65

Key words: Geophysical fluid dynamics, climate dynamics, low-frequency variability, attractor bifurcation, dynamic transition, well-posedness.

Contents

*Corresponding author. *Email addresses:* ljp@lasg.iap.ac.cn (J. P. Li), showang@indiana.edu (S. H. Wang)

1 Introduction

The atmosphere and ocean around the earth are rotating geophysical fluids, which are also two important components of the climate system. The phenomena of the atmosphere and ocean are extremely rich in their organization and complexity, and a lot of them cannot be produced by laboratory experiments. The atmosphere or the ocean or the couple atmosphere and ocean can be viewed as an initial and boundary value problem (Bjerknes [5], Rossby [125], Phillips [120]), or an infinite dimensional dynamical system. These phenomena involve a broad range of temporal and spatial scales (Charney [11]). For example, according to J. von Neumann [147], the motion of the atmosphere can be divided into three categories depending on the time scale of the prediction. They are motions corresponding respectively to the short time, medium range and long term behavior of the atmosphere. The understanding of these complicated and scientific issues necessitate a joint effort of scientists in many fields. Also, as John von Neumann [147] pointed out, this difficult problem involves a combination of modeling, mathematical theory and scientific computing.

In this article, we shall address mathematical and numerical issues in geophysical fluid dynamics and climate dynamics. The main topics include:

1. issues on the modeling, mathematical analysis and numerical analysis of the primitive equation (PEs),
2. climate variability, predictability and successive bifurcation,
3. a new dynamical systems theory and its applications to geophysical fluid dynamics.

As we know, the atmosphere is a compressible fluid and the seawater is a slightly compressible fluid. The governing equations for either the atmosphere, or the ocean, or the coupled atmosphere-ocean models are the general equations of hydrodynamic equations together with other conservation laws for such quantities as the energy, humidity and salinity, and with proper boundary and interface conditions. Most general circulation models (GCMs) are based on the PEs, which are derived using the hydrostatic assumption in the vertical direction. This assumption is due to the smallness of the aspect ratio (between the vertical and horizontal length scales). We shall present a brief survey on recent theoretical and computational developments and future studies of the PEs.

One of the primary goals in climate dynamics is to document, through careful theoretical and numerical studies, the presence of climate low frequency variability, to verify the robustness of this variability's characteristics to changes in model parameters, and to help explain its physical mechanisms. The thorough understanding of this variability is a challenging problem with important practical implications for geophysical efforts to quantify predictability, analyze error growth in dynamical models, and develop efficient forecast methods. As examples, we discuss a few sources of variability, including wind-driven (horizontal) and thermohaline (vertical) circulations, El Niño-Southern Oscillation (ENSO), and Intraseasonal oscillations (ISO).