Numerical Study of the Unsteady Aerodynamics of Freely Falling Plates

Changqiu Jin\textsuperscript{1} and Kun Xu\textsuperscript{2,*}

\textsuperscript{1} Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, Beijing 100088, China.
\textsuperscript{2} Mathematics Department, Hong Kong University of Science and Technology, Hong Kong, China.

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Abstract. The aerodynamics of freely falling objects is one of the most interesting flow mechanics problems. In a recent study, Andersen, Pesavento, and Wang [J. Fluid Mech., vol. 541, pp. 65-90 (2005)] presented the quantitative comparison between the experimental measurement and numerical computation. The rich dynamical behavior, such as fluttering and tumbling motion, was analyzed. However, obvious discrepancies between the experimental measurement and numerical simulations still exist. In the current study, a similar numerical computation will be conducted using a newly developed unified coordinate gas-kinetic method [J. Comput. Phys, vol. 222, pp. 155-175 (2007)]. In order to clarify some early conclusions, both elliptic and rectangular falling plates will be studied. Under the experimental condition, the numerical solution shows that the averaged translational velocity for both rectangular and elliptical plates are almost identical during the tumbling motion. However, the plate rotation depends strongly on the shape of the plates. In this study, the details of fluid forces and torques on the plates and plates movement trajectories will be presented and compared with the experimental measurements.

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1 Introduction

It is an observed fact that not all falling objects travel straight downward under the influence of gravity. For examples, leaves, tree seeds, and paper cards all follow complicated downward trajectories as they fall under gravity. In fact, falling leaves and tumbling sheets of paper often reverse their downward direction, momentarily rising against
gravity, as they flutter or tumble through the air. It is therefore clear that any satisfactory explanation of this complicated natural phenomenon must include a description of the instantaneous fluid forces experienced by the falling object as well as the inertial and gravitational effects that are present.

Previously, only a limited number of analytical results have been proven with reference to objects falling through fluid under gravity, and those that do exist most commonly address only special limits, such as Stokes flow and inviscid irrotational flow. Most falling objects, however, encounter unsteady aerodynamic forces as they fall. As a result, the problem of a rigid body falling through a viscous fluid has recently attracted some attention, especially with the help of computational fluid dynamics. An excellent investigation has conducted by Pesvento and Wang [13], where in which the governing Navier-Stokes equations were solved numerically in the frame of the falling object.

The fluttering, looping, and tumbling motions have been of interest to physicists since the 19th century, when famed Scottish physicist James Clerk Maxwell studied falling cards and offered a qualitative explanation of the correlation between the sense of rotation and the drift direction of a tumbling card [10], when the classical aerodynamic theory had not been established yet. In the past decades, most investigation only presented qualitative or average properties, such as the phase diagram, where the instantaneous fluid forces were not obtained [2, 4, 5, 14, 15]. Recently, Mittal, Seshadri and Udaykumar (2004) in [11] solved two-dimensional Navier-Stokes equations for a freely falling cylinder. Jones and Shelley [9] suggested a falling card model based on inviscid theory and the unsteady Kutta condition.

Wang and Pesavento [13] studied the aerodynamics of freely falling plates for a quasi two-dimensional flow at Reynolds numbers around $10^3$, which is a typical state for a leaf or business card falling in air. They measured the plates trajectories experimentally using a high speed digital video at sufficient resolution and determined the instantaneous plates' acceleration, from which they obtained the instantaneous fluid force and torque on the falling objects. Furthermore, besides the experimental measurements, the direct numerical solutions of the two-dimensional incompressible Navier-Stokes equations for the falling objects have been obtained. The trajectories and forces on the moving plates from both numerical computation and experimental measurement have been qualitatively compared. The discrepancies, such as the falling trajectories and angular velocities, were ascribed to the differences in geometries between the rectangular cross-section in the experiment and the elliptical one in numerical simulation.

In this study, we will investigate the motion of freely falling plates numerically using a newly developed gas-kinetic scheme, in which the governing Navier-Stokes equations are solved on a moving grid. In the computations that follow, the grid will be fixed to the falling plate and so the motion of the grid itself will be determined by the translational and rotational motion of the plate. Similar to the Lagrangian method, each grid will follow its own grid velocity during a time step, but the flow update in the whole computational domain around the falling plate is in an common inertial reference of frame. Therefore, there will have no additional forces related to the non-inertial frame of ref-