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Parallel Algebraic Multigrid Methods in Gyrokinetic Turbulence Simulations

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Abstract. Parallel algebraic multigrid methods in gyrokinetic turbulence simulations are presented. Discretized equations of the elliptic operator $-\nabla^2 u+\alpha u=f$ (with both $\alpha=0$ and $\alpha\neq 0$) are ubiquitous in magnetically confined fusion plasma applications. When α is equal to zero a "pure" Laplacian or Poisson equation results and when α is greater than zero a so called Helmholtz equation is produced. Taking a gyrokinetic turbulence simulation model as a testbed, we investigate the performance characteristics of basic classes of linear solvers (direct, one-level iterative, and multilevel iterative methods) on 2D unstructured finite element method (FEM) problems for both the Poisson and the Helmholtz equations.

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1 Introduction

The largest unknown in designing a tokamak reactor is the turbulent plasma transport. Plasma particles and heat escape much faster than the time scale predicted by the classical binary collision model. Turbulence in plasma inherently differs from those in neutral fluids, in that the interaction of charged particles and the electromagnetic waves plays an important role as the instability drive as well as the turbulence regulation (or the dissipation) mechanism.

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One of the popular methods in simulating fully ionized plasmas is the particle in cell (PIC) method [15]. The PIC codes evolve plasma dynamics self-consistently by alternately *pushing* charged particles and *solving the fields* that are the set of Maxwell's equations for the electromagnetic fields. The PIC method retains important kinetic effects such as nonlinear wave-particle interactions (nonlinear Landau damping), which cannot be captured by the fluid models [12,23]. While traditionally PIC methods were restricted to local phenomena, the invention of the gyrokinetic simulation method [29] enabled us to study plasma turbulence in global scales. The basic idea behind the gyrokinetic simulation method is to time-average rapid precessing motions, and only to push the guiding center motion for the particles. Instead, the finite Larmor radius effects enter the system through the *gyrokinetic Poisson equation* [29,30]. The gyrokinetic Poisson equation is in an integral form and is solved with a linear iterative method [34], under the condition when the electrons are adiabatic [16]. Unfortunately, this iterative method fails in the presence of electromagnetic effects or shear Alfven dynamics [6]. In the latter case the iteration matrix is no longer diagonally dominant and a new linear solution method is required.

The particle in cell codes keep the numbers of particles per cell nearly constant [15]. Consequently global PIC code (GTC for example) in toroidal geometry employs a logically non-rectangular grid with a number of poloidal grid points increasing in the radial direction. In this work, a finite element method (FEM) is employed for the elliptic solver [37]. In general, the FEM is suitable for dealing with complicated geometries, where unstructured meshes are employed. For example, for the International Thermonuclear Experimental Reactor (ITER) size plasma [27], where the minor radius is on the order of one thousand ion Larmor radii, several million grid points per poloidal plane are needed. For the practical application for the ITER size burning plasmas the time taken by the field solver becomes significant if naive solvers are employed. Thus, time taken for the field solver becomes an issue in plasma simulations (fluid or kinetic) with larger the grid number [42]. Successful preconditioning of the linear solvers is a cornerstone of the gyrokinetic turbulence simulations. In this paper we introduce algebraic multigrid (AMG) methods and demonstrate its efficiency for the plasma physics application. We would like to note that the FEM solver and AMG developed in this work is also applicable for a general particle in cell code [15] as well as Vlasov type simulation method [13].

The field solve entails the solution of an elliptic operator of the form

$$-\nabla^2 u+\alpha u=f, \tag{1.1}$$

where α is equal to zero for "pure" Poisson problems and is greater than zero for the so called Helmholtz problems. As we discuss below, in our gyrokinetic simulation, u represents the field quantities such as Φ (electrostatic potential) and $A_{\|}$ (vector potential), while f represents the source terms such as the charge density and the current density. This field solve is often discretized with an unstructured FEM, resulting in a symmetric positive definite system of linear algebraic equations.

The earliest methods for solving the sparse linear systems of equations that arise in finite difference techniques and the finite element method, are of two types: direct methods