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MHD Turbulence Studies using Lattice Boltzmann Algorithms

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Abstract. Three dimensional free-decaying MHD turbulence is simulated by lattice Boltzmann methods on a spatial grid of 8000^3 for low and high magnetic Prandtl number. It is verified that $\nabla \cdot B = 0$ is automatically maintained to machine accuracy throughout the simulation. Isosurfaces of vorticity and current show the persistence of many large scale structures (both magnetic and velocity) for long times — unlike the velocity isosurfaces of Navier-Stokes turbulence.

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1 Introduction

Here we examine free decaying 3D magnetohydrodynamics (MHD) by a mesoscopic algorithm that, unlike standard computational fluid dynamic (CFD) algorithms, is amenable

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to massive parallelization [1]. Indeed, our lattice Boltzmann (LB) code [1] has had a sustained performance of 26.25 TFlops/s on 4800 PEs of the *Earth Simulator* — i.e., 67% of peak and outputting 0.25 TB of data. Moreover, the $\nabla\cdot\mathbf{B}=0$ constraint is automatically enforced, thus side-stepping the need for divergence cleaning. Our work is a generalization of the seminal 2D LB-MHD algorithm of Dellar [2].

The basic idea behind the LB method [3,4] is to project the desired nonlinear macroscopic system into a higher dimensional phase space with the resulting kinetic system simpler to solve and readily parallelized. The difficult nonlinear convective derivatives $\mathbf{u}\cdot\nabla\mathbf{u}$, $\mathbf{u}\cdot\nabla\mathbf{B},\cdots$ (where $\mathbf{u}$ is the fluid velocity and $\mathbf{B}$ the magnetic field) of CFD are now replaced by simple linear advection (a shift operation) and local collisional relaxation in phase space. On performing the Chapman-Enskog long-time long-wavelength asymptotics [2–4] on the discretized LB system, one recovers the MHD equations to leading order in the Knudsen number and thus relating the MHD transport coefficients to the relaxation parameters in the BGK collision operators of LB. The essential point is that nonlocal macroscopic gradients, like the mean strain rate or $\nabla\cdot\mathbf{B}$, are computed at the mesoscopic LB level by simple local moments of the distribution functions. To recover [2–4] the Navier-Stokes equation, one need only introduce a scalar distribution function $f(\mathbf{x},\xi,t)$ whose zeroth moment yields the density and first moment yields the momentum. The importance of Dellar's work [2] was his introduction of a vector distribution function $\mathbf{g}(\mathbf{x},\xi,t)$ whose zeroth moment defines the magnetic field $\mathbf{B}$.

One minimizes the computational memory requirements resulting from the transformation from $(\mathbf{x},t)$- to $(\mathbf{x},\xi,t)$-space by a clever choice of discretization of ξ-space. In particular, it has been shown [3,4] that one can recover the 3D Navier-Stokes equation with a 15-bit discretization of ξ-space. One must also consider the numerical stability of LB — especially as one pushes to smaller and smaller transport coefficients — since LB is an explicit, second order accurate scheme. In its simplest formulation [3,4], there are no constraints imposed on the discretized velocity distribution function $f_\alpha(\mathbf{x},t)$ to maintain its positive definiteness throughout the simulation. Recently this problem has been successfully addressed for the Navier-Stokes equation [5–11] by imposing an entropy constraint on the discretized $f_\alpha(\mathbf{x},t)$ to enforce positive-definiteness. This has resulted in an entropic lattice Boltzmann (ELB) scheme that is unconditionally stable. An outstanding problem is whether a similar ELB scheme can be devised to LB MHD.

In Section 2, we briefly introduce the LB and ELB schemes for Navier-Stokes turbulence and introduce the lattice discretization of ξ-space by a 15-, 19- or 27-velocities at each spatial node. We then introduce our 3D LB MHD representation. The parallelization and performance our LB schemes on various supercomputer architectures is discussed in Section 3. In Section 4 we first present some of our basically fully resolved ELB simulations for Navier-Stokes turbulence on a $1600\times1600\times1600$ spatial grid at a Reynolds number of 25000. These simulations clearly indicate intermittency [12] in the turbulence by the deviation of the energy spectrum from the $k^{-5/3}$ Kolmogorov spectrum. In Section 4.2, we present LB-MHD simulations on a $1800\times1800\times1800$ spatial grid for magnetic Prandtl number $\mathrm{Pr}=0.3$ and $\mathrm{Pr}=3.0$, where Pr is the ratio of the viscosity