

Two-Relaxation-Time Lattice Boltzmann Scheme: About Parametrization, Velocity, Pressure and Mixed Boundary Conditions

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Received 14 Mar 2007; Accepted (in revised version) 22 July 2007

Communicated by Shiyi Chen

Available online 9 October 2007

Abstract. We develop a two-relaxation-time (TRT) Lattice Boltzmann model for hydrodynamic equations with variable source terms based on equivalent equilibrium functions. A special parametrization of the free relaxation parameter is derived. It controls, in addition to the non-dimensional hydrodynamic numbers, any TRT macroscopic steady solution and governs the spatial discretization of transient flows. In this framework, the multi-reflection approach [16, 18] is generalized and extended for Dirichlet velocity, pressure and mixed (pressure/tangential velocity) boundary conditions. We propose second and third-order accurate boundary schemes and adapt them for corners. The boundary schemes are analyzed for exactness of the parametrization, uniqueness of their steady solutions, support of staggered invariants and for the effective accuracy in case of time dependent boundary conditions and transient flow. When the boundary scheme obeys the parametrization properly, the derived permeability values become independent of the selected viscosity for any porous structure and can be computed efficiently. The linear interpolations [5, 46] are improved with respect to this property.

PACS: 47.10.ad, 47.56+r, 02.60-x

Key words: Lattice Boltzmann equation, Dirichlet boundary conditions, Chapman-Enskog expansion, multiple-relaxation-time model, BGK model, TRT model, Navier-Stokes equation, Stokes equation, recurrence equations, staggered invariants.

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1 Introduction

The two-relaxation-time (TRT) Lattice Boltzmann model [17–20] is suitable to model solutions of the Navier-Stokes and hyperbolic non-linear advective-diffusion equations. A very simple linear collision TRT operator is based on the decomposition of the population solution into its symmetric and anti-symmetric components. Two locally prescribed eigenvalues (relaxation parameters) determine the evolution of the symmetric and anti-symmetric collision components. When the eigenvalues are equal, the TRT reduces to the BGK operator [41] (called sometimes *single-relaxation-time*, SRT). The SRT and TRT solutions can be obtained using the multiple-relaxation-times (MRT) collision operator [15, 27–31, 36]. At second order, the incompressible macroscopic mass and momentum conservation equations are identical for TRT, SRT and MRT but these models differ for higher-order approximations.

For TRT and SRT, both the kinematic and bulk viscosity are related to the selected “symmetric” eigenvalue and the coefficients of the diffusion tensor are related to the prescribed “anti-symmetric” eigenvalue. The effective values of the transport coefficients depend also on the distribution of the equilibrium components between the different velocities: the hydrodynamic equations are usually modeled with isotropic equilibrium weights [41] whereas the anisotropic diffusion tensors need anisotropic ones (see [17, 19]). The TRT model enables a very simple analysis of its solutions based on parity arguments. One example presents the multi-reflection (MR) boundary schemes, first developed in [16] for the Navier-Stokes equation and then adapted in [18] for the symmetric equilibrium components, e.g., any scalar diffusion variable. Another example presents the analysis of the interface conditions in [20], suitable for flat interface between two immiscible fluids and for modeling Darcy flows in heterogeneous stratified soils.

There are strong numerical evidences (see [16]) that the permeability of an arbitrary porous media is independent of the chosen viscosity value when a specific combination of symmetric/anti-symmetric eigenvalues (so-called “magic parameter”, here Λ_{eo}) is fixed and no-slip conditions are modeled either with the bounce-back or with one particular multi-reflection (MR1) scheme [16]. In other words, the obtained momentum distribution for Stokes flow, multiplied with the modeled viscosity, depends on the selected eigenvalues only via their combination Λ_{eo} . This property implies a very specific functional dependency of the coefficients on the eigenvalues for all the terms in the population expansion and boundary rules. The earlier exact solution [13] confirmed this for parabolic flow. In this paper we give a rigorous explanation of these observations for arbitrary flow, owing to the derivation in [32] of *equivalent* link-wise recurrence equations. They allow to demonstrate that any steady non-dimensional velocity and pressure TRT solution, obtained with the Stokes or Navier-Stokes equilibrium functions, is governed by Λ_{eo} on a given grid, in addition to the Reynolds, Froude and Mach numbers. We will show that bounce-back and MR1 share this property exactly.

In contrast with the SRT, the TRT can keep Λ_{eo} at any prescribed value for both hydrodynamic and convective-diffusion equations when their transport coefficients vary. This