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High Performance Algorithms Based on a New Wavelet Expansion for Time Dependent Acoustic Obstacle Scattering

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Abstract. This paper presents a highly parallelizable numerical method to solve time dependent acoustic obstacle scattering problems. The method proposed is a generalization of the "operator expansion method" developed by Recchioni and Zirilli [SIAM J. Sci. Comput., 25 (2003), 1158-1186]. The numerical method proposed reduces, via a perturbative approach, the solution of the scattering problem to the solution of a sequence of systems of first kind integral equations. The numerical solution of these systems of integral equations is challenging when scattering problems involving realistic obstacles and small wavelengths are solved. A computational method has been developed to solve these challenging problems with affordable computing resources. To this aim a new way of using the wavelet transform and new bases of wavelets are introduced, and a version of the operator expansion method is developed that constructs directly element by element in a fully parallelizable way. Several numerical experiments involving realistic obstacles and "small" wavelengths are proposed and high dimensional vector spaces are used in the numerical experiments. To evaluate the performance of the proposed algorithm on parallel computing facilities, appropriate speed up factors are introduced and evaluated.

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1 Introduction

In this paper, we present a new version of the operator expansion method that improves the method developed in [1]. Roughly speaking, the operator expansion method is used to solve an exterior boundary value problem for the Helmholtz equation by reducing it to a sequence of systems of first kind integral equations defined on a suitable reference surface. Three innovations are introduced:

1) the use of the wavelet transform in a way different from that suggested in [1] that allows us to compute the coefficient matrices of the systems of linear equations mentioned above element by element in a fully parallelizable way;

2) new bases of wavelets with an (arbitrary) assigned number of vanishing moments that generalize the Haar's basis;

3) the representation on these wavelet bases of the integral operators, the unknowns and the data of the systems of integral equations obtained with the operator expansion method in order to approximate the integral equations with sparse systems of linear equations.

These innovations make the development of a highly parallelizable numerical method possible to deal with scattering problems involving obstacles having complex shapes and wavelengths small when compared to the characteristic dimensions of the obstacles. That is, problems that require the use of a large number of unknowns and equations can be discretized satisfactorily. In fact, thank to the representation of the integral operators on the wavelet bases and to a simple truncation procedure, matrices that approximate the integral operators with a very high sparsity factor are obtained. Consequently, high dimensional problems in the discretized variables can be solved at an affordable computational cost.

Let us begin by introducing some notation. Let $\mathbb{R}$ be the set of real numbers, $\mathbb{R}^h$ be the h-dimensional real Euclidean space, and $\underline{x}=(x_1,x_2,\cdots,x_h)^T\in\mathbb{R}^h$ a generic vector. Let $(\cdot,\cdot)$ and $\|\cdot\|$ denote the Euclidean scalar product and the corresponding Euclidean vector norm respectively.

Let $\mathbf{C}$ be the complex numbers. For $z\in\mathbf{C}$ we denote with $Re(z)$ and $Im(z)$ the real and imaginary parts of z respectively. We denote with $\mathbf{C}^h$ the h dimensional complex Euclidean space.

Let $\Omega\subset\mathbb{R}^3$ be a bounded simply connected open set with locally Lipschitz boundary $\partial\Omega$ and let $\overline{\Omega}$ be its closure. Furthermore we denote with $\underline{n}(\underline{x})=(n_1(\underline{x}),n_2(\underline{x}),n_3(\underline{x}))^T\in\mathbb{R}^3$ the outward unit normal vector to $\partial\Omega$ in $\underline{x}\in\partial\Omega$. We note that when $\partial\Omega$ is a locally Lipschitz surface the unit normal vector $\underline{n}(\underline{x})$ exists almost everywhere in $\underline{x}$ when $\underline{x}\in\partial\Omega$ (see [2, Theorem 1.8 p. 17 and p. 52]).

We assume that $\partial\Omega$ can be decomposed as follows: $\partial\Omega=\partial\Omega_1\cup\partial\Omega_2$, where $\partial\Omega_1$, $\partial\Omega_2$ are two locally Lipschitz surfaces such that $\partial\Omega_1\cap\partial\Omega_2=\emptyset$. Finally we assume that $\partial\Omega_1$ is characterized by a boundary acoustic impedance given by a bounded continuous non-negative real function $\chi=\chi(\underline{x})$, $\underline{x}\in\partial\Omega_1$, and that $\partial\Omega_2$ is characterized by an infinite