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The Method of Fundamental Solutions for Steady-State Heat Conduction in Nonlinear Materials

A. Karageorghis[1,*] and D. Lesnic[2]

[1] *Department of Mathematics and Statistics, University of Cyprus, 1678 Nicosia, Cyprus.*
[2] *Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, UK.*

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Abstract. The steady-state heat conduction in heat conductors with temperature dependent thermal conductivity and mixed boundary conditions involving radiation is investigated using the method of fundamental solutions. Various computational issues related to the method are addressed and numerical results are presented and discussed for problems in two and three dimensions.

AMS subject classifications: 65N35, 65N38, 65H10

Key words: Nonlinear heat conduction, radiation, method of fundamental solutions.

1 Introduction

Two-dimensional boundary value problems of heat conduction in nonlinear materials and nonlinear boundary conditions have been investigated using the boundary element method (BEM) by Bialecki and Nowak [3] and Ingham *et al.* [12]. However, the implementation of the BEM becomes rather tedious for problems in three-dimensional irregular domains. Moreover, the evaluation of the gradient of the temperature solution on the boundary requires the use of finite differences or the evaluation of hypersingular integrals. In order to alleviate some of these difficulties, this paper proposes the use of the method of fundamental solutions (MFS), a meshless Trefftz-type method which is considerably easier to implement. The advantages of the MFS over the finite difference method (FDM), the finite element method (FEM), and the BEM for solving elliptic boundary value problems, especially in higher-dimensions where no discretization of the solution domain, or its boundary, is necessary, are well-documented, see for example the survey papers [6,7,10].

*Corresponding author. *Email addresses:* `andreask@ucy.ac.cy` (A. Karageorghis), `amt5ld@maths.leeds.ac.uk` (D. Lesnic)

The MFS was first applied to potential flow problems by Johnston and Fairweather [13] and has since been applied to a large variety of physical problems. In this work, we shall employ the same idea of expressing the solution of the Laplace equation as a linear combination of fundamental solutions with singularities located outside the domain of the problem under consideration. In [14], Karageorghis and Fairweather used the MFS for solving linear material problems with nonlinear radiative boundary conditions. The purpose of this study is to extend this analysis to nonlinear material problems in two and three dimensions.

The mathematical formulation of the problem is given in Section 2 and the MFS description in Section 3. In the previous study of Karageorghis and Fairweather [14], the gradient of the nonlinear least-squares objective function which is minimized was calculated internally by default using 'blind' finite differences. Thus with perturbing the parameters one at a time, the Jacobian matrix is recalculated at every iteration. Therefore, the finite-difference approach for calculating the gradient has a high computational cost, see Rus and Gallego [25]. In order to save on the computational time the Jacobian matrix is calculated analytically. Numerical results are compared with the BEM results of Bialecki and Nowak [3] for two test examples in Section 4. Moreover, a three-dimensional example is considered, apparently, for the first time. Finally, comments and conclusions are presented in Section 5.

2 Mathematical formulation

We consider a simply-connected bounded domain $\Omega \subset \mathbb{R}^d$, $d \geq 2$, with piecewise smooth boundary $\partial\Omega$ and assume that this boundary is composed of three disjoint parts Γ_1, Γ_2 and Γ_3. On each part $\Gamma_i, i=1,2,3$ boundary conditions of the first (Dirichlet), second (Neumann) and third (Robin) kind, respectively, hold. The mathematical problem governing steady-state heat conduction is given by, see [3],

$$\nabla \cdot (k(T)\nabla T) = 0 \quad \text{in} \quad \Omega, \tag{2.1}$$

subject to the boundary conditions

$$T = f \quad \text{on} \quad \Gamma_1, \tag{2.2a}$$

$$-k(T)\frac{\partial T}{\partial n} = g \quad \text{on} \quad \Gamma_2, \tag{2.2b}$$

$$k(T)\frac{\partial T}{\partial n} + h\left[T - T_f\right] + C_0 R\left[T^4 - T_s^4\right] = q \quad \text{on} \quad \Gamma_3, \tag{2.2c}$$

where T is the temperature solution, k is the thermal conductivity, $\boldsymbol{n}$ is the unit outward normal vector to the boundary $\partial\Omega$, f is a prescribed temperature on the boundary Γ_1, g is a prescribed heat flux on the boundary Γ_2 and q is a given function on the boundary Γ_3 which is usually taken to be zero. Also, h is the convective heat transfer coefficient, T_f is