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Spatial Correlation Function in Modular Networks

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Abstract. Due to the complexity of the interactions among the nodes of the complex networks, the properties of the network modules, to a large extent, remain unknown or unexplored. In this paper, we introduce the spatial correlation function G_{rs} to describe the correlations among the modules of the weighted networks. In order to test the proposed method, we use our method to analyze and discuss the modular structures of the ER random networks, scale-free networks and the Chinese railway network. Rigorous analysis of the existing data shows that the spatial correlation function G_{rs} is suitable for describing the correlations among different network modules. Remarkably, we find that different networks display different correlations, especially, the correlation function G_{rs} with different networks meets different degree distribution, such as the linear and exponential distributions.

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Key words: Correlation function, weighted networks, modular structure.

1 Introduction

In many real world networks, it was found that there exist significant subnetworks (so-called network modules) in their structures, such as the metabolic networks [1], food webs [2], social networks [3] and Internet [4]. Many theoretical and experimental results indicate that the network modules perform specific tasks in the functional properties of such networks. A major current challenge is to understand their topological structures and the roles they playing in the networks. What are the mechanisms by which network modules emerge in the network? How to describe the interactions among the network modules?

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Recently, a number of methods have been proposed to understand the structure properties of complex networks, such as the statistical, mathematical and model-based analysis methods. Newman and Girvan proposed the clustering algorithm in [5, 6], where the quantitative definition of modularity was firstly introduced. In [7], the network is mapped into a spin system. Here, the problem of finding the modularity of a network is analogous to the standard statistical mechanism problem. Donetti and Muñoz proposed an algorithm that combines spectral methods with clustering techniques [8]. Ziv, Middendorf and Wiggins outlined an information theoretical algorithm by applying the information bottleneck on probability distributions [9]. In [10], a weighted network is proposed to model and simulate the Dutch railway network. For a review, see [11, 12].

Correlation is an important property of networks, which is of special interest [13–15]. For example, whether gregarious people are more likely to contact with gregarious people? or whether an old web site is more likely to connect to old ones? These questions often have fundamental importance in reality. The correlation function provides a tool to give a better insight into the problem mentioned above. In this paper, we introduce a new correlation function and use it to capture an explicit and obvious relationship between the network modules and that have highly connected nodes. The main aim is to get a better understanding of the network modules and their correlations. The paper is organized as follows: we introduce the proposed method in Section 2; the numerical and analytical results are presented in Section 3; finally, conclusions are presented.

2 The proposed method

The appearance of the network modules represents a broad range of natural phenomena. To describe these phenomena requires an understanding of the basic topological structures of such networks. These topological structures are based on the links of the nodes. A reasonable assumption is that the correlations among the network modules are describe by the links of the nodes on a coarse-grained level. In this paper, to assume that the modular structures of networks have been determined in advance, we describe the properties of the network modules and their correlations by introducing the spatial correlation function G_{rs}.

Our method is as follows. A weighted network which has N nodes is considered. Let w_{ij} be the weight of the edge that links the node i and the node j. We introduce the spatial correlation function $G_{rs}(i)$. Assume that the node i is within the module r and the node j is within the module s. Then the definition of $G_{rs}(i)$ is as follows:

$$G_{rs}(i) = \frac{1}{n_s}\sum_{j=1}^{N_s} d_i d_j w_{ij}, \tag{2.1}$$

where n_s represents the number of the links between the node i and the nodes within the module s, N_s represents the number of the neighbours of the node i and d_i is the degree of the node i. The degree d_i of the node i is defined as the number of edges linked to the