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A Spectral Element Implementation for the M3D Extended MHD Code

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Abstract. A spectral element library has been developed and integrated with the M3D extended MHD code. The currently used linear triangular finite element implementation and the new high order quadrilateral spectral element implementation are directly compared on equilibrium, linear stability, and nonlinear evolution calculations run on the same problems.

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1 Introduction

Spectral element methods offer several possible advantages for MHD simulations. They are high order discretizations and offer the possibility of exponential decrease of the error with increasing degree. Since spectral element methods can be implemented with discrete operators that are combinations of tensor product matrices and point-wise operations, they can be implemented efficiently at close-to-peak on modern computer architectures. The resulting global stiffness and system matrices are sparse block matrices in which the blocks are dense and of a special structure. Direct solvers can use static condensation and sparse solvers for the much smaller Schur complement system which leads to a fast and efficient solution algorithm.

M3D [1,2] is a highly modular code for extended MHD problems. Its modularity allows the implementation of several discretizations and the change from one to another for the same problem. Originally, M3D used a spectral discretization in the toroidal and poloidal angles combined with finite differences in the radial direction. While leading to fast solvers and accurate solutions, that discretization did not allow for complicated

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geometries of the cross sections such as needed to model stellarators and divertor tokamaks. To handle such geometries, a discretization with linear finite elements (FEM) [3] was introduced, which is still the standard version at present. The implementation of a spectral element discretization, described in this presentation, should allow for complex geometries while at the same time recovering the high accuracy that the original version achieved on simple geometries.

Spectral element (SEL) methods [4–6] have been recently introduced in MHD simulations [7,8]. The SEL approach offers several possible benefits. The discretization can be made accurate to high order, with exponential decrease of error as the order is increased. Static condensation provides an efficient solution method for elliptic problems, in which the Schur complement matrix to be solved is orders of magnitude smaller than the original matrix. Curved isoparametric elements allow alignment with relatively complicated boundary shapes encountered in simulation of magnetic fusion experiments. Spectral elements give a diagonal mass matrix, which is advantageous for the partially implicit M3D time stepping scheme.

Against these benefits, there is a concern that high order methods are only good for smooth problems and will not work for highly nonlinear turbulent MHD flows which can occur in magnetic fusion disruptions and Edge Localized Modes (ELMs).

The implementation of M3D presented here solves exactly the same equations, using the same top level code, with either FEM or SEL discretizations. This makes it possible to compare the two methods. This paper presents a direct comparison of MHD equilibrium, linear stability, and nonlinear evolution calculations using the FEM and SEL discretizations in M3D.

A spectral element discretization of M3D has been parallelized using OpenMP for shared memory computers, and MPI/PETSc for distributed memory computers. The OpenMP version is more restricted in problem size, mainly because no domain decomposition in poloidal planes is employed. Each poloidal plane is assigned to a separate processor. In M3D, all the elliptic solves such as Poisson's equation are two dimensional, and do not couple the poloidal planes. Parallelization consists of solving each poloidal plane simultaneously, giving a linear scaling with number of processors. The MPI/PETSc distributed memory spectral element version is being developed. For now, the standard PETSc solvers are used, without taking advantage of static decomposition. We would expect that parallel scaling with number of processors should scale similarly to the scaling of the present distributed memory M3D implementation. The use of static decomposition would be expected to improve performance of the solvers. This will be addressed in the future.

2 Spectral elements

The implemented SEL elements have C^0 continuity, which is the standard approach. There are C^1 SEL methods, but they are more complicated to implement and have certain