COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. 2, No. 4, pp. 746-759

Commun. Comput. Phys.
August 2007

Implementation of the Divergence-Free and Pressure-Oscillation-Free Projection Method for Solving the Incompressible Navier-Stokes Equations on the Collocated Grids

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Received 14 June 2006; Accepted (in revised version) 23 November 2006

Available online 22 January 2007

Abstract. In this paper, a divergence-free and pressure-oscillation-free projection method for solving the incompressible Navier-Stokes equations on the non-staggered grid is presented. The exact discrete projection method is used to compute the velocity field, which ensures the discrete divergence of the velocity field is zero. In order to eliminate the odd-even decoupling in the pressure field, a filtering procedure is proposed and applied to the pressure field. We have shown this filter recovers the grid scale ellipticity in the pressure field and the odd-even decoupling can be removed effectively. The proposed numerical scheme is further verified through numerical experiments.

AMS subject classifications: 65M06, 76D05, 65M15

Key words: Incompressible Navier-Stokes equations, projection methods, non-staggered grids, pressure filtering.

1 Introduction

Conventional numerical methods for solving the incompressible Navier-Stokes (N-S) equations in terms of the primitive variables are mainly applied on the marker-and-cell (MAC) [13] type staggered grids. For this type of grids, the pressure, density and other scalars are stored in the mesh cell center, the velocities are stored at the mesh cell faces and the momentum equations are solved by constructing separate control volumes around them. This arrangement makes the stencil for the pressure gradient terms in the

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momentum equations very compact. The continuity equation can be also computed directly requiring no interpolations. These two features make the staggered grid approach capable of capturing all resolvable modes and thereby preventing the odd-even decoupling or the "checkerboard" modes of the pressure field.

However, the use of the staggered grids adds geometrical complexity by the introduction of multiple control volumes. Furthermore, the staggered grid schemes become very awkward when generalizing to curvilinear meshes and unstructured grids that are commonly used to handle the complex geometries. The use of staggered grids for complex geometries leads to either high memory requirements, or inefficient solution methods or complicated equations with additional source terms [27].

An alternative form of the control volume and fluid variable positioning technique, namely the non-staggered or collocated grid arrangement, stores all the variables at the same physical location and employs only one set of control volumes. This approach reduces the geometrical complexity and shortens the long computational time needed in the conventional staggered methods and is becoming increasingly popular in practical applications.

A significant shortcoming of the non-staggered grid approach is the so called odd-even coupling phenomenon or the occurrence of "checkerboard" modes in the pressure field. When the NS equations are solved by the projection methods [5-8,14-20,23,25], this phenomenon has also been observed [22].

There are two types of projection methods, namely the "exact" discrete projection methods and the approximate projection methods [1]. In the exact projection method, the discrete Poisson equation is defined as the product of the discrete divergence operator and the discrete pressure gradient operator, and the discrete divergence of velocity is zero, or more precisely, small quantity within the convergence tolerance of the solution of the discrete Poisson equation. When second-order central difference approximations are implemented for both operators, the discrete Poisson equation corresponds to a non-compact sparse stencil and produces an oscillatory pressure field [22].

If the discrete Poisson equation is derived through a straightforward discretization of the continuous Laplacian operator, the resulting projection scheme is called the approximate projection method [1]. In this case, the discrete divergence of velocity is not zero, but is rather a function of the truncation error. For the pressure-free projection methods, the approximate projection procedure can effectively remove the checkerboard modes in the pressure field [27]. However, for the incremental-pressure projection method, it is generally not sufficient to eliminate the pressure oscillations [16]. Therefore, other supplementary measurements must be used. These measurements include the momentum interpolation technique [21] and its variants [3,4,9], various filters designed by Lai [16] and Riders [22], and the fourth-order "compact equivalent" approximation of the discrete Poisson equation [10].

Although the approximate projection methods perform well in many practical applications, they inevitably produce velocity fields that are not divergence-free in the discrete sense. The non-divergence-free velocity fields sometimes have adverse effects on the ac-