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Simulations of Compressible Two-Medium Flow by Runge-Kutta Discontinuous Galerkin Methods with the Ghost Fluid Method

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Abstract. The original ghost fluid method (GFM) developed in [13] and the modified GFM (MGFM) in [26] have provided a simple and yet flexible way to treat two-medium flow problems. The original GFM and MGFM make the material interface "invisible" during computations and the calculations are carried out as for a single medium such that its extension to multi-dimensions becomes fairly straightforward. The Runge-Kutta discontinuous Galerkin (RKDG) method for solving hyperbolic conservation laws is a high order accurate finite element method employing the useful features from high resolution finite volume schemes, such as the exact or approximate Riemann solvers, TVD Runge-Kutta time discretizations, and limiters. In this paper, we investigate using RKDG finite element methods for two-medium flow simulations in one and two dimensions in which the moving material interfaces is treated via non-conservative methods based on the original GFM and MGFM. Numerical results for both gas-gas and gas-water flows are provided to show the characteristic behaviors of these combinations.

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Key words: Runge-Kutta discontinuous Galerkin method, WENO scheme, ghost fluid method, approximate Riemann problem solver.

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1 Introduction

In this paper, we investigate and compare between the RKDG finite element methods employed for two-medium flow simulations in which the different non-conservative methods based on the original ghost fluid method (GFM) [13] and the modified GFM (MGFM) [26] are used to treat the moving material interface.

The Runge-Kutta discontinuous Galerkin (RKDG) method [8–12], for solving hyperbolic conservation laws given as

$$\begin{cases} U_t+\nabla\cdot F(U)=0 \\ U(x,0)=U_0(x), \end{cases} \tag{1.1}$$

is a high order finite element method employing the useful features from high resolution finite volume schemes, such as the exact or approximate Riemann solvers, TVD Runge-Kutta time discretizations [35, 36], and total variation bounded (TVB) limiters [34] or weighted essential non-oscillatory (WENO) type limiter [31, 32]. RKDG methods have been widely applied and perform very well to solve for single-medium compressible flow problems.

On the other hand, a relatively dominant difficulty for simulating compressible two-medium flow is the treatment of the moving material interfaces and their immediate vicinities. There can arise severe nonphysical oscillations in the vicinity of the material interface especially in the presence of shock and large density ratio even when a well-established numerical method for single-medium flow is applied directly to the multi-medium flow. As such, there are numerous works published in the literature on how to overcome this difficulty [1, 2, 4, 7, 20, 22, 24].

In the fairly recent times, the ghost fluid method (GFM) as proposed by Fedkiw et al. [13] has provided an alternative and yet flexible way to treat the two-medium flow. The main characteristic of the GFM is its simplicity, ease of extension to multi-dimensions and maintenance of a sharp interface without smearing. Essentially, the GFM makes the interface "invisible" during calculations by defining ghost cells and ghost fluids, and the ensuring computations are then carried out as for a single-medium manner via solving two respective single-medium GFM Riemann problems. As such, its extension to multi-dimensions becomes fairly straightforward. Also, since only single-fluid flux formulations are required for the GFM, the said method can be employed for any two fluids of vastly different EOS (equation of state), compressible-incompressible or viscous-inviscid two-fluid flow [5]. There is subsequently developed variations of the original GFM [14] with other applications as can be found in [2, 21].

However, it is precisely the manner of treatment of the single medium across the interface in the GFM that may cause numerical inaccuracy in the presence of a strong shock wave interacting with the interface [26]; this is especially so if such wave interaction with the interface is not taken into account properly in the definition of ghost fluid states. This happens because the dynamics of shock refraction at the material interface and the resultant interfacial status are highly dependent on the material properties on both sides