COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **5**, No. 2-4, pp. 515-531

Commun. Comput. Phys.
February 2009

Numerical Solution of Partial Differential Equations in Random Domains: An Application to Wind Engineering

Claudio Canuto* and Davide Fransos

Dipartimento di Matematica, Politecnico di Torino, C.so Duca degli Abruzzi 24, 10129 Torino, Italy.

Received 10 October 2007; Accepted (in revised version) 31 December 2007

Abstract. An application of recent uncertainty quantification techniques to Wind Engineering is presented. In particular, the study of the effects of small geometric changes in the Sunshine Skyway Bridge deck on its aerodynamic behavior is addressed. This results in the numerical solution of a proper PDE posed in a domain affected by randomness, which is handled through a mapping approach. A non-intrusive Polynomial Chaos expansion allows to transform the stochastic problem into a deterministic one, in which a commercial code is used as a black-box for the solution of a number of Reynolds-Averaged Navier-Stokes simulations. The use of proper Gauss-Patterson nested quadrature formulas with respect to a Truncated Weibull probability density function permits to limit the number of these computationally expensive simulations, though maintaining a sufficient accuracy. Polynomial Chaos approximations, statistical moments and probability density functions of time-independent quantities of interest for the engineering applications are obtained.

AMS subject classifications: 65C20, 65N35

Key words: Uncertainty quantification, stochastic partial differential equations, random domains, mapping approach, polynomial chaos, wind engineering.

---

1 Introduction

Wind Engineering models incorporate various parameters which are or potentially may be affected by uncertainty; they concern both the fluid-dynamic and the structural components of each model. The study of the wind field has indeed been approached over the years by statistical models based on observation data. On the other hand, until recently,

---

*Corresponding author. *Email addresses:* `claudio.canuto@polito.it` (C. Canuto), `davide.fransos@polito.it` (D. Fransos) the study of the aerodynamic and aeroelastic behavior of structures has been mostly accomplished by deterministic approaches. Yet, geometric uncertainties may dramatically influence the reliability of the model. For instance, as in Aerospace Engineering [11], random discrepancies between the ideal geometries conceived in the design phase and their actual realization tested in wind tunnels may lead to significant variations in the resulting flow field. The development of efficient tools for the uncertainty quantification of the response of structures is therefore an important task for Wind Engineering.

From the mathematical point of view, geometric uncertainties lead to the formulation of boundary value problems in random domains. As an alternative to expensive Monte-Carlo techniques, the transformation of such problems into deterministic ones by an appropriate change of unknowns allows their discretization by standard numerical methods. Karhunen-Loeve (KL) and Polynomial Chaos (PC) expansions [9, 19] provide the mathematical ground for these transformations. Different methodologies have been explored in the recent literature for handling the randomness of the domain; they include the mapping of each random domain to a reference domain [4, 11, 14, 20], the inclusion of the random domains into a fictitious domain to which the PDE is extended [3], the solution of the deterministic PDEs satisfied by the moments of the variables of interest [10]. In the Wind Engineering applications the number of the random variables is usually fairly small (thus the use of sparse methods is not mandatory). On the other hand, the computational cost of a single simulation on a given realization of the random domain is so high that non-intrusive/collocation methods are practically unavoidable. Within them, the selection of the interpolation/collocation grids is critical to achieve an acceptable balance between cost and accuracy.

This paper is a step of an ongoing project aiming at describing a set of stochastic effects on bridges [6]. Here we isolate a single input random variable related to the bridge deck cross-section (the curvature radius of the lower surface's corners) and we investigate its effects on the statistics of some relevant time-averaged integral quantities of the flow.

We compute the approximate PC with respect to a Truncated Weibull probability density function, and the associated family of nested Gauss-Patterson formulas. They are used in the reconstruction of finite PC expansions from the outputs of a limited number of Reynolds-Averaged Navier-Stokes simulations.

We give an application to the case study of the Sunshine Skyway Bridge deck. Two incident turbulence regimes give rise to two different patterns of the PC expansion. In both cases, sufficiently accurate PC approximations are obtained.

2 Mathematical setting

In order to present the mathematical setting that will be used in the Wind Engineering application, we first consider a linear scalar model, which is representative of some of the relevant features of the subsequent model without being affected by its complications.

Let $(\Omega,\mathcal{F},\mathcal{P})$ be a probability space, where Ω is the state space, $\mathcal{F}$ is the σ-algebra of