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A Macro Projective Integration Method in $2D$ Microscopic System Applied to Nonlinear Ion Acoustic Waves in a Plasma

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Abstract. In the *Equation-free* framework, a macro-coarse projective integration method consists of two parts: the time stepper and time projection on macro scale. The first one consists of lifting, micro simulation and restriction. For extracting directly from microscopic simulations the information which would be obtained from the macroscopic model of two-dimensional microscopic systems, the time stepper based on the one-dimensional cumulative distribution functions, the marginal cumulative and appropriate number of the conditional cumulative distributions, is introduced. Here this procedure is tested on the nonlinear ion acoustic wave in a plasma. The numerical micro-solver is the one dimensional electrostatic particle-in-cell code. It is shown that particle correlations related to wave structures are better preserved by the new model. The lifting step is critically related to the noise in system. The enlarged noise, rise of correlations, trapping of particles during the wave steepening can seriously violate the basic assumptions of the equation-free approach.

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1 Introduction

The macroscopic, e.g., coherent behavior in the complex systems emerges in the interactions of microscopic constituents-atoms, molecules, cells, individuals of a population-among themselves and with an environment. As a consequence the macroscopic behavior can somehow be deduced from the microscopic one. For some problems like

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Newtonian fluid mechanics the Navier-Stokes equation predated its microscopic derivation from the kinetic theory. However, in many problems in chemistry, ecology,material science, engineering, etc. the closures required to translate them from the microscopic (particle) level to a high-level macroscopic description are unknown. Severe limitations arise in trying either to find closures or to solve these problems at the scale at which the questions of interest are asked, by using microscopic simulations only. The *Equation-free* (EFREE) proposed by I.G. Kevrekidis et al. [1–4] is one of the systematic frameworks for directly extracting from microscopic simulations the information which would be obtained from macroscopic models had these been available in a closed form. This is a system based procedure processing the results of short bursts of appropriately initialized microscopic simulations.

The main tool that allows the performance of numerical tasks at the macroscopic level using the microscopic (e.g., stochastic) simulation codes is the so-called *coarse time-stepper*. It consists of three parts: lifting (mapping from coarse-macroscopic to microscopic level), short time micro calculations around which the macroscopic calculations are wrapped and restriction (mapping from fine-micro scale to macroscopic level) [1]. The details about each part are presented in many papers [1–10]. The coarse time stepper is combined with time projection at macroscopic level, i.e., time projection of the coarse observables on the macroscopic scale. Significant premise in the EFREE framework is the clear separation between micro and macro time scales.

The complexity of plasma phenomena challenged researchers to try to implement the multiscale approaches developed in other scientific fields. One of these attempts is the implementation of the EFREE procedure by Shay et al. [11], in the context of the nonlinear ion-acoustic wave as the preparatory step for the intriguing task to solve a problem of the magnetic reconnection. There, the ion-acoustic wave propagation and steepening are originally followed by the modified three-dimensional electromagnetic particle-in-cell (3*D* EM PIC) code. In EFREE the electrons are adiabatic, both the electron and ion velocity distributions are assumed to be the shifted Maxwellian and quasineutrality is proposed. The results of the multiscale EFREE calculations are discussed with respect to the full micro-PIC simulations. At the first step the coarse observables are determined. First three moments: ion density, ion velocity and pressure are taken as the'active' coarse observables, i.e., those macro variables which are directly computed forward in time. On the other hand, the electron density, electron velocity and electric field are taken as the 'passive' coarse variables, i.e., variables which are not calculated directly but from active macro observables. These observables are defined on the coarse mesh by the linear interpolation procedure [11]. The micro quantities, the ion and electron positions are obtained through the *lifting* from corresponding densities and the ion and electron velocities are lifted from corresponding velocity distributions (approximated by the shifted Maxwellian). The PIC solver is then applied for the short time in order to ensure the system to stay near the so-called slow manifold. In other words, the implementation of the micro solver has to ensure the reconstruction of the values of the macro quantities which would be obtained under the same conditions but using only the micro solver. The