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An Improved Tokamak Sawtooth Benchmark for 3D Nonlinear MHD

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Abstract. Accurate prediction of the sawtooth cycle [1] is an important test for nonlinear MHD codes. The sawtooth cycle in the CDX-U tokamak [2], chosen because its small size and low temperature allow simulation using actual device parameters, has been an important benchmark for the comparison of the M3D [3] and NIMROD [5] codes for the last several years. Successive comparisons have led to improvements and refinements in both codes. The most recent comparisons show impressive agreement between the two codes both on the linear instability and on the details of nonlinear cyclical behavior. These tests are somewhat idealized and do not yet agree quantitatively with the experimentally observed sawtooth period. We expect a second generation of CDX-U sawtooth benchmarks based on an analytically specified equilibrium, with source terms that show greater fidelity to the physical device, to produce better agreement.

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1 Introduction

Verification and validation of 3D nonlinear MHD initial value codes is a particularly challenging task. The inherent high sensitivity of nonlinear systems to small differences in initial conditions makes it difficult to distinguish the effects of differences in representation or time-advance scheme from differences in fidelity to the physical model when making detailed comparisons of the predictions of two different codes for a particular instability or other event. It is still more difficult to compare such predictions directly

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with experimental observations, for which measurement error leads to far greater uncertainty regarding the initial conditions. Nevertheless, such efforts are necessary to justify confidence in the predictive capabilities of these codes.

A program of verification and validation has been undertaken for the two workhorse 3D MHD codes of the SciDAC Center for Extended MHD Modeling (CEMM) [4], M3D [3] and NIMROD [5]. The nonlinear instability chosen for the test is the resistive internal kink mode that gives rise to the sawtooth crash [1], a fundamental dynamic of the inductive tokamak discharges these codes are intended to model. The crash involves magnetic reconnection across a thin helical current sheet, a structure whose size varies roughly inversely with the plasma temperature. For large magnetic fusion experiments, the high temperatures result in a current sheet too thin to be practically resolved by present-day codes. Some small tokamaks, however, such as CDX-U [2], are cold enough to have resolvable current layers and hence make good targets for validation studies using actual device parameters. CDX-U was thus chosen for this study.

In this article, we present results of the first successful CEMM cross-code nonlinear verification benchmark: the CDX-U sawtooth cycle. Because this problem should also be of value to the larger MHD modeling community, we also propose a new version of the benchmark with an analytically specified initial state. Preliminary results with this new equilibrium are then presented.

2 Statement of the problem

The CDX-U tokamak is a small ($R_0 = 33.5$ cm), low-aspect-ratio ($R_0/a = 1.5$) device with a typical operating temperature of about $T_e = 100$ eV. Modeling 3D macroscopic activity in the experiment requires the specification of an initial equilibrium as well as sources and transport coefficients. The initial equilibrium configuration and sources are provided by running the 2D transport timescale code TSC [6] to match typical traces of the plasma current $I_p(t)$ from the experiment. A sequence of experimentally relevant equilibria, each at a fixed time, are obtained from the TSC computation as described in [7]. We note that as the central current density increases in the TSC calculation, the central safety factor q_0, a measure of the pitch of the local magnetic field, falls below unity, the condition for onset of the resistive internal kink instability. A single kink-unstable TSC equilibrium is then chosen to be used as the initial condition for a complete run of each of the two 3D nonlinear codes. For the initial benchmark, we initialized the 3D codes with an equilibrium in which $q_0 = 0.92$.

Both M3D and NIMROD are parallel 3D nonlinear magnetohydrodynamic (MHD) codes in toroidal geometry, solving a superset of the resistive MHD equations that describe the behavior of a collisional magnetized plasma on timescales long compared to electrostatic oscillations but typically short compared to resistive diffusion. The equa-