

Numerical Diffusion Control of a Space-Time Discontinuous Galerkin Method

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Abstract. Variations on space-time Discontinuous Galerkin (STDG) discretization associated to Runge-Kutta schemes are developed. These new schemes while keeping the original scheme order can improve accuracy and stability. Numerical analysis is made on academic test cases and efficiency of these schemes are shown on propagating pressure waves.

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1 Introduction

Controlling the numerical diffusion with an upwinding technique is not a new idea, but this task is rather difficult when solving the nonlinear equations of gas dynamics for compressible flows with a DG approach [2–5]. The numerical flux in space (for example the Roe flux or the Lax-Friedrichs flux) are fixed once and for all and there are no parameters to be tuned except those of the Runge-Kutta time scheme.

What we propose here in this paper is to devise variants of the DG approach. The space-time DG approach (STDG) [7–9] leads to naturally implicit schemes solved iteratively. Our first variant is to use a truncated explicit process to replace the iterative implicit solver (STDG-RK). The second variant STDG- α consists in an adapted upwinding in time of the STDG scheme when computing the time fluxes. This scheme can improve convergence for steady flows as a higher CFL can be used in the pseudo-time solver. A third variant (RKDG-NDC) consists in upwinding the Runge-Kutta space DG approach, leading to a unified formulation with the iterative implicit STDG scheme. These schemes allow control of numerical diffusion.

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A numerical study of precision and stability is presented for the 1D linear advection equation and the Burgers equation both for steady and unsteady problems.

DG is a nonlinear scheme. To preserve monotonicity, limiters should be used when solving nonlinear problems. But in many cases (nonlinear aeroacoustics, subsonic, transonic and vortical flows for example), they are not necessary. In this paper, all computations are done without limiters. MUSCL results are shown to provide reference results as the classical DG can be seen as an extension of the MUSCL approach [1].

Results are shown on the academic test case of a planar wave propagating upstream a subsonic flow. The RKDG-NDC result is compared to a computation without numerical diffusion control. As will be seen, the RKDG-NDC shows very little diffusion without loss of accuracy or stability.

Another result concerns the study of a planar acoustic wave interacting with a circular temperature spot on a Cartesian mesh. This study is connected to high frequency combustion instability [11] This aeroacoustic application is displayed associated with an AMR technique [12–14].

These formulations are easily extended to curvilinear grid as shown in the case of a transonic flow around the NACA0012 airfoil.

2 Numerical discretization

Following the works of many authors (see for example Cockburn [6] or van der Vegt [7–9]), DG is now standard for solving conservation law equations. This paper is a sequel of [3] in which different classical space DG variants were compared to MUSCL. We look here at a space-time DG formulation which gives better results than previously for nearly the same computational effort. In our implementation of RKDG (Space) or STDG (Space-Time) formulations in our Euler solver, only a $P1$ approximation has been used (but $P1$ and $P2$ accuracy tests have been conducted on a 1D scalar equation). The Euler implementation has been realized within an AMR platform [12].

2.1 Governing equations

We consider the Euler equations written in the compact conservative form:

$$\partial_t W + \vec{\nabla} \cdot \mathbf{F}(W) = \mathbf{0}, \quad (2.1)$$

where W is the conservation variable vector:

$$W = (\rho, \rho \vec{U}, \rho E), \quad (2.2)$$

and $\mathbf{F} = (f, g, h)^T$ the flux vector:

$$\begin{cases} f = (\rho u, \rho u^2 + p, \rho uv, \rho uw, u(\rho E + p)), \\ g = (\rho v, \rho uv, \rho v^2 + p, \rho vw, v(\rho E + p)), \\ h = (\rho w, \rho uw, \rho vw, \rho w^2 + p, w(\rho E + p)), \end{cases} \quad (2.3)$$