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Segmentation of Images from Fourier Spectral Data

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Abstract. This paper designs a segmentation method for an image based on its Fourier spectral data. An edge map is generated directly from the Fourier coefficients without first reconstructing the image in pixelated form. Consequently the internal boundaries of the edge map are not blurred by any (filtered) Fourier reconstruction. The edge map is then processed with an edge linking segmentation algorithm. We include examples from magnetic resonance imaging (MRI). Our results illustrate some potential benefits of using high order methods in imaging.

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Key words: Fourier coefficients, edge detection, segmentation.

1 Introduction

Fourier spectral data is often the source of image information. For example, magnetic resonance imaging (MRI) and synthetic aperture radar (SAR) sensors measure the values of the Fourier coefficients of an image. Techniques such as the filtered Fourier reconstruction are then used to create the image. In many instances, clinicians and diagnosticians are not interested in the detailed structure of the image, but rather the shapes that outline a specific region of interest. Typically in such cases the Fourier coefficients are used to reconstruct the image at specified pixel points so that a standard segmentation algorithm, e.g. [7,19,22,23,25,26], can be applied. There are clear downsides to this approach, however. Most notably, since the image to be segmented is necessarily only piecewise smooth, the reconstruction is polluted by the Gibbs ringing artifact at the jump discontinuities (edges) of the image, [6,8,17,21], making it difficult to determine where the boundaries of internal regions lie. Smoothing filters are used to reduce the oscillations, but have the undesirable consequence of "blurring" the edges over several pixel points. In [2], the

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Gegenbauer reconstruction algorithm, [18], was adapted to resolve a brain MRI free from Gibbs ringing and without blurring the boundaries of each tissue region. Consequently, standard segmentation algorithms used to extract features of interest returned dramatically improved results, [1]. Unfortunately high order reconstruction methods, such as the Gegenbauer reconstruction method, can be cost prohibitive for large data sets. This investigation therefore adopts a different strategy based on the following observations: (i) image reconstruction is not necessary for region extraction; (ii) without using costly reconstruction algorithms, converting the Fourier coefficients into a pixelated image either produces Gibbs oscillations that clutter the edges, or damping, which may blur the edges beyond recognition; and (iii) vital information about the internal boundaries of an image stored in the Fourier data can be easily extracted. Hence we seek to develop a segmentation algorithm that directly uses Fourier information to extract the internal edges of an image. We emphasize the distinction between our proposed method, which starts with the Fourier coefficients of an image, and standard segmentation algorithms, which start with pixelated data.

Once a high quality edge map of the image is generated, we design a segmentation algorithm that links the points on the edge map to create closed contour regions. Specifically, the algorithm produces sequences of ordered pairs which in turn can be used as an initial guess in algorithms that parameterize contour regions for each feature of interest, [27]. We note that the intensity values of our edge map approximate the magnitude of the jump discontinuities of the image, and *not* the underlying image values at the internal boundaries. Although we are unable to determine a-priori error estimates, our examples demonstrate the techniques proposed here provide a good starting point for designing fast segmentation algorithms from Fourier data, and therefore illustrate the benefits of using high order methods in imaging.

This paper is organized as follows. In Section 2 we review the concentration method developed in [13–15] which locates jump discontinuities (edges) in piecewise smooth functions directly from Fourier spectral data. The method is extended to two dimensions in Section 3 to produce the edge map of an image. In Section 4 we present our segmentation algorithm that generates (closed) parameterized contours from the edge map. The output can now be used to initiate contour minimization routines which can accurately estimate and extract particular features of interest, [9, 27]. We summarize our results in Section 5 and discuss possible future applications.

2 Edge detection from Fourier spectral data

Suppose we are given the Fourier spectral data of an image. In order to extract any particular feature of interest, we must first be able to visualize its internal boundaries. One way to do this is to first reconstruct the image on a prescribed set of pixels, for instance by using filtered Fourier algorithms, [21]. However, as stated in the introduction, this type of projection often causes blurring at the internal boundaries, making segmentation