COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. 4, No. 5, pp. 1129-1150

Commun. Comput. Phys.
November 2008

Progress of Pattern Dynamics in Plasma Waves

B. Qiao[1,2,3], C. T. Zhou[3,4,5,*], X. T. He[3,4,5] and C. H. Lai[2,6]

[1] *Center for Plasma Physics, Department of Physics and Astronomy, Queen's University Belfast, Belfast BT7 1NN, UK.*
[2] *Department of Physics, National University of Singapore, Singapore 117542.*
[3] *Institute of Applied Physics and Computational Mathematics, P. O. Box 8009, Beijing 100088, China.*
[4] *Center for Applied Physics and Technology, Peking University, Beijing 100871, China.*
[5] *Institute for Fusion Theory and Simulation, Zhejiang University, Hangzhou 310027, China.*
[6] *Beijing-Hong Kong-Singapore Joint Centre for Nonlinear & Complex Systems (Singapore), National University of Singapore, Singapore 117542.*

Received 29 February 2008; Accepted (in revised version) 8 July 2008

Available online 9 September 2008

Abstract. This paper is concerned with the pattern dynamics of the generalized nonlinear Schrödinger equations (NSEs) related with various nonlinear physical problems in plasmas. Our theoretical and numerical results show that the higher-order nonlinear effects, acting as a Hamiltonian perturbation, break down the NSE integrability and lead to chaotic behaviors. Correspondingly, coherent structures are destroyed and replaced by complex patterns. Homoclinic orbit crossings in the phase space and stochastic partition of energy in Fourier modes show typical characteristics of the stochastic motion. Our investigations show that nonlinear phenomena, such as wave turbulence and laser filamentation, are associated with the homoclinic chaos. In particular, we found that the unstable manifolds $W^{(u)}$ possessing the hyperbolic fixed point correspond to an initial phase $\theta=45^\circ$ and 225°, and the stable manifolds $W^{(s)}$ correspond to $\theta=135^\circ$ and 315°.

PACS: 47.54.-r, 05.45.Yv, 52.25.Gj, 52.35.Mw

Key words: Pattern dynamics, homoclinic chaos, nonlinear Schrödinger equations, plasma waves.

1 Introduction

It is well known that the generalized nonlinear Schrödinger equation (NSE) [1–25] is of the form

$$iE_t+\partial_x^2 E+F(|E|^2)E=0, \tag{1.1}$$

*Corresponding author. *Email addresses:* b.qiao@qub.ac.uk (B. Qiao), zcangtao@iapcm.ac.cn (C. T. Zhou), xthe@iapcm.ac.cn (X. T. He), phylaich@nus.edu.sg (C. H. Lai)

where $E(x,t)$ is the complex amplitude of waves, and t and x are time and space variables, respectively. The function F is used to describe different physical processes [20–23], such as plasma physics, nonlinear optics, fluid dynamics, superconductivity theory and Bose-Einstein condensates (BEC) etc. The generalized NSE is one of the basic evolution models for nonlinear process in various branches of the conservative systems. Early applications of the NSE were in the context of nonlinear optics where it described the propagation of light beams in nonlinear media [24]. Also it has been applied to gravity waves on deep water, for which the predicted modulational instability and envelope soliton formation have been clearly demonstrated experimentally [25]. In plasma physics, a large number of nonlinear processes, such as nonlinear hydromagnetic waves [26], small-K condensation of weak turbulence in nonlinear plasmas, Langmuir waves in electrostatic plasmas, femtosecond (fs) laser pulse in air, medium-intensity laser in underdense plasma and intense laser pulse in relativistic plasmas, can all be effectively modeled by the generalized NSE (1.1) with different potential function F.

For the generalized NSE (1.1), the Lagrangian density is

$$L=\frac{i}{2}(E^*E_t-EE_t^*)-|E_x|^2+f(|E|^2), \tag{1.2}$$

where $f(|E|^2)=\int_0^{|E|^2}F(s)ds$. The system described by it is a conservative system. According to the Norther theorem, we can obtain the following invariants: the quasiparticle number

$$N=\int |E|^2dx, \tag{1.3}$$

the momentum

$$P=i\int E^*E_xdx, \tag{1.4}$$

and the Hamiltonian quantity

$$H=\int [|E_x|^2-f(|E|^2)]dx. \tag{1.5}$$

For such a conservative Hamiltonian system (1.1) or (1.2), the nonlinear dynamics including solitary waves and patterns are very important. In nature, most nonlinear phenomena, such as Langmuir wave collapse, laser self-focusing and filamentation, are all associated with the basic nonlinear dynamics of the system and are the results of nonlinear development of modulational instability. However, the latter has not been systematically studied, while previous work has concentrated on the solitary wave solutions and the singular solutions. The study of complex dynamics including chaos and patterns for the generalized NSE (1.1) associated with different physical problems is of interest to the understanding of various nonlinear phenomena in plasmas.