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Magnetohydrodynamics with Implicit Plasma Simulation

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Abstract. We consider whether implicit simulation techniques can be extended in time and space scales to magnetohydrodynamics without any change but the addition of collisions. Our goal is to couple fluid and kinetic models together for application to multi-scale problems. Within a simulation framework, transition from one model to the other would occur not by a change of algorithm, but by a change of parameters. This would greatly simplify the coupling. Along the way, we have found new ways to impose consistent boundary conditions for the field solver that result in charge and energy conservation, and establish that numerically-generated stochastic heating is the problem to overcome. For an MHD-like problem, collisions are clearly necessary to reduce the stochastic heating. Without collisions, the heating rate is unacceptable. With collisions, the heating rate is significantly reduced.

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Key words: Plasma simulation, MHD, particle-in-cell, implicit methods.

1 Introduction

Plasma dynamics span a large range of time and space scales, and no one model can cover them all. For very fast time scales and very small spatial scales, classical plasma simulations work very well, but at the cost of resolving all scales and consuming vast amounts of computing power. Magnetofluid (MHD) calculations model large scales and long times, but eliminate kinetic effects. In the landscape between, there are reduced models, such as hybrid and gyrokinetic models, and there are implicit methods, which

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extend plasma simulations to larger scales while retaining the contributions of kinetic electrons. An increase in applicability of any of these methods would be useful.

Here we ask if implicit methods can be extended to problems on MHD time scales. Do the equations become so inaccurate and difficult to solve that there is no value in doing so? Specifically, we ask if the field equations become substantially more difficult to solve, and whether stochastic heating can be controlled as one increases time and length scales. There is a recent review of implicit simulation [23], to which we direct readers wishing a more comprehensive review. On the other hand, we include some details that have not previously been reported in order to proceed with our analysis.

Implicit simulation methods were invented by Mason [25] and Denavit [13], who combined fluid and kinetic models of electrostatic plasmas to bypass stability constraints imposed by explicit-in-time methods. Explicit stability constraints require that

$$\omega_{pe}\Delta t < 2, \quad h/(u_e/\omega_{pe}) < 1,$$

and that $c\Delta t/h < 1$ for electromagnetic plasma interactions, where ω_{pe} is the electron plasma frequency, c is the speed of light, Δt is the time step, and h is the minimum resolved length scale. The minimum resolved scale, h, must be less than the electron Debye length, u_e/ω_{pe}, to avoid the finite grid instability [18,24]. Mason and Denavit noted the solution of the field equations requires only the first few moments of the particle distribution, and that the evolution of the moments can be predicted using a much smaller system of equations than is required to advance the particle orbits. By solving implicit-in-time moment and field equations self-consistently, the implicit moment method advances the particle solutions just once each cycle, as in an explicit solution, while retaining the superior numerical stability properties of a fully implicit method.

The moment equations are derived from an expansion of the moments about their initial values in powers of $u_e\Delta t/h$, where u_e is the root mean square electron speed. The accuracy of the expansion requires that $u_e\Delta t/h < 1$, but

$$u_e\Delta t/h = (u_e/\omega_{pe})/h \times \omega_{pe}\Delta t.$$

Thus Δt can be increased if h is also increased. h can be increased because implicit methods are less prone to the finite-grid-instability than explicit methods [3], and Δt can be increased because of the unconditional stability of implicit methods. The advantage of the implicit solution is most evident in the scaling of the cost of a simulation with the ion/electron mass ratio. Given a problem on ion time and space scales with

$$T = n\omega_{pi}^{-1}, \quad L = l(c/\omega_{pi}),$$

the explicit/implicit cost ratio scales as $(m_i/m_e)^{3/2}$. Data on the relative cost of explicit and implicit calculations in 2 dimensions is given in Table 1 of [31]. It should be noted that the explicit calculations resolve all scales, and the implicit calculations do not. There is much more detail in the explicit results. On the other hand, explicit calculations cost