COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **3**, No. 3, pp. 704-723

Commun. Comput. Phys.
March 2008

Stability Analysis of the Immersed Boundary Method for a Two-Dimensional Membrane with Bending Rigidity

Zhaoxin Gong[1], Huaxiong Huang[1,2,*] and Chuanjing Lu[1]

[1] *Department of Mechanics, Shanghai Jiaotong University, Shanghai 200240, China.*
[2] *Department of Mathematics and Statistics, York University, Toronto, Ontario M3J 1P3, Canada.*

Received 19 January 2007; Accepted (in revised version) 5 September 2007

Available online 27 November 2007

Abstract. In this paper, we analyze the stability of the Immersed Boundary Method applied to a membrane-fluid system with a plasma membrane immersed in an incompressible viscous fluid. We show that for small deformations, the planar rest state is stable for a membrane with bending rigidity. The smoothed version, using a standard regularization technique for the singular force, is also shown to be stable. Furthermore, we show that the coupled fluid-membrane system is stiff and smoothing helps to reduce the stiffness. Compared to the system of elastic fibers immersed in an incompressible fluid, membrane with bending rigidity consist of a wider range of decay rates. Therefore numerical instability could occur more easily for an explicit method when the time step size is not sufficiently small, even though the continuous problem is stable.

AMS subject classifications: 74K15, 76D05, 35Q30, 76D27, 76M25, 65M06

Key words: Bending rigidity, immersed boundary method, membrane, moving interface, stability.

1 Introduction

In nature as well as in engineering applications, there exist abundant examples where a flexible structure is immersed in a viscous incompressible fluid. Such a structure-fluid interaction is especially relevant in biological systems. The immersed boundary method, which was developed by Peskin [1] and Peskin and McQueen [2] to study the nature of the blood flow in the heart, is an effective technique for modeling and simulating this type

*Corresponding author. *Email addresses:* jackyff@sjtu.edu.cn (Z. X. Gong), hhuang@mathstat.yorku.ca (H. X. Huang), cjlu@sjtu.edu.cn (C. J. Lu)

of fluid-structure interactions. The immersed boundary method considers the structure as an immersed boundary, which can be represented by a singular force in the Navier-Stokes equations rather than a real body. It avoids difficulties associated with moving boundaries faced by conventional methods. The immersed boundary method is both a mathematical formulation and a numerical scheme. The mathematical formulation employs a mixture of Eulerian and Lagrangian variables. These are related by interaction equations in which the Dirac delta function plays a prominent role. The numerical implementation consists of several steps. First of all, the Eulerian (field) variables are defined on a fixed Cartesian mesh while the Lagrangian variables (related to the immersed boundary) are defined on a curvilinear grid that lies on top of the fixed Cartesian mesh. Secondly, a smoothing (discretization) is needed for the Dirac delta function, constructed according to certain principles, e.g., by matching the moments. The immersed boundary method has been applied to a variety of problems, such as the swimming of eels, sperm and bacteria [3–5], ameboid deformation [6], platelet aggregation during blood clotting [7, 8], and the deformation of red blood cells in a shear flow [9].

Despite the popularity of the immersed boundary method as a computational tool, only few analysis have been given of the method itself. Beyer and LeVeque [10] provided one of the first convergence analysis using a one-dimensional model. Tu and Peskin [11] performed stability analysis for three different methods including the immersed boundary method for solving fluid flow problems with moving interfaces. Stockie [12] and Stockie and Wetton [13, 14] presented a linear stability analysis on both continuous and discrete versions of the immersed boundary method applied to fluid flows with immersed fibers. All analysis above assumed that the immersed structure was elastic without bending resistance. On the other hand, biological cell membranes are nearly incompressible with bending rigidity. Thus bending resistance cannot be ignored in flow problems involving biological cells, especially when the curvature of the immersed boundary is large. Our aim in this paper is to analyze the stability of an immersed boundary with bending resistance moving in an incompressible viscous fluid.

We consider a two-dimensional membrane immersed in a quiescent flow field at the equilibrium state. We formulate the problem using the immersed boundary approach and carry out linear stability analysis by computing the eigenvalues of the membrane-fluid system in a periodic box. The smoothed version is considered next by regularizing the Dirac delta function, following Peskin [1]. Without regularization (smoothing), the singular delta function leads to jumps in the pressure and the velocity gradient. One can use these jump conditions to construct numerical method, such as the immersed interface method proposed by LeVeque and Li [15] and analyzed by Huang and Li [16]. Therefore, the analysis of the unsmoothed version is more relevant to the immersed interface method while that of the smoothed version is directly related to the immersed boundary method used in [1–9]. Our analysis shows that both versions are linearly stable but the membrane with bending rigidity is more stiff than the (linear) elastic membrane-fluid system. Thus, the system with bending resistive membrane is more difficult to solve from computational point of view.