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Simulation of MHD Flows Using a Hybrid Lattice-Boltzmann Finite-Difference Method

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Abstract. A hybrid lattice-Boltzmann finite-difference method is presented to simulate incompressible, resistive magnetohydrodynamic (MHD) flows. The lattice Boltzmann equation (LBE) with the Lorentz force term is solved to update the flow field while the magnetic induction equation is solved using the finite difference method to calculate the magnetic field. This approach is methodologically intuitive because the governing equations for MHD are solved in their respective original forms. In addition, the extension to 3-*D* is straightforward. For validation purposes, this approach was applied to simulate the Hartmann flow, the Orszag-Tang vortex system (2-*D* and 3-*D*) and the magnetic reconnection driven by doubly periodic coalescence instability. The obtained results agree well with analytical solutions and simulation results available in the literature.

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Key words: Lattice Boltzmann method, LBM, hybrid, finite difference method, Magnetohydrodynamics, MHD.

1 Introduction

In recent years, the lattice-Boltzmann method (LBM) has experienced enormous success in the simulations of various flow problems [1, 2] and attempts have been made to develop LBM algorithms for MHD problems. Chen *et al*. [3] and Martinez *et al*. [4] employed the bidirectional streaming for 2-*D* MHD problems where the distribution function is propagated into two different directions associated with the velocity and magnetic field. The former used 37 discrete velocities while the latter reduced it to 13. Schaffenberger and Hanslmeier [5] later reduced the number of velocities even further to nine by employing the standard streaming rule on a 2-*D* square lattice. Dellar [6] developed a new method, where two distribution functions are utilized to represent the hydrodynamic

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momentum and the magnetic induction. The hydrodynamic part is simulated using the conventional low Mach number LBM, and the magnetic field is represented by a separate vector-valued magnetic distribution function, which obeys the vector Boltzmann-BGK equation. This method has been later extended to 3-D by Breyiannis and Valougeorgis [7].

In all the afore-mentioned methods, the magnetic induction problem as well as the flow problem is dealt with by a lattice kinetic approach. While employing LBM for the Boltzmann equation is natural, the use of a kinetic approach for solving the magnetic induction equation is not quite intuitive because, after all, the Boltzmann equation and the magnetic induction equation constitute a set of governing equations for MHD. In other words, a lattice kinetic approach does not need to be used always for the magnetic induction problem even though all-kinetic approaches are more consistent and have many advantages in many cases. In fact, other numerical methods, such as the finite difference method, can be easily employed to solve the magnetic induction equation with equal or better accuracy because those methods are well established.

In this article, the authors present an alternative hybrid method, where the flow field is obtained by LBM and the magnetic induction equation is solved by a finite difference method. Therefore, the fundamental governing equations for MHD are solved without introducing a lattice kinetic approach in the calculation of the magnetic field. This approach can be easily extended to 3-D. In this study, this method is applied to Hartmann flow, Orszag-Tang vortex system (both 2-D and 3-D) and magnetic reconnection problem for validation purposes. The obtained results agree well with analytical solutions and the numerical solutions available in the literature.

2 Mathematical model

The Boltzmann transport equation with the Bhatnagar-Gross-Krook (BGK) collision term is written as:

$$\frac{\partial f}{\partial t}+\xi\cdot\nabla_x f+a\cdot\nabla_\xi f=-\frac{f-f^{eq}}{\lambda}, \tag{2.1}$$

where $f=f(x,\xi,t)$ is the single-particle distribution function in both physical space and phase space, x is the position vector, ξ is the microscopic velocity, a is the acceleration due to the external force exerting on the particles, λ is the relaxation time due to collisions and f^{eq} is the equilibrium distribution function, which is described by the Maxwell-Boltzmann distribution as follows:

$$f^{eq}=\frac{\rho}{(2\pi RT)^{D/2}}exp\left[-\frac{(\xi-u)^2}{2RT}\right], \tag{2.2}$$

where ρ,u,T,R,D are density, macroscopic velocity, temperature, gas constant and dimension of space respectively. For MHD flows, the acceleration a can be written as:

$$a=\frac{1}{\rho\mu}(\nabla\times B)\times B, \tag{2.3}$$