COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. **5**, No. 2-4, pp. 620-634

Commun. Comput. Phys.
February 2009

Analysis and Computation for a Fluid Mixture Model

Qunlei Jiang, Zhilin Li* and Sharon R. Lubkin

Department of Mathematics & Center for Scientific Computations, North Carolina State University, Raleigh, NC 27695, USA.

Received 30 September 2007; Accepted (in revised version) 18 January 2008

Available online 1 August 2008

Abstract. A fluid mixture model of tissue deformations has been studied in this paper. The model is a mixed system of nonlinear hyperbolic and elliptic partial differential equations. Both theoretical linear stability and numerical analysis are presented. Comparisons between standard numerical methods that utilize Runge-Kutta methods coupled with the WENO scheme and the immersed interface methods are given. Numerical examples are also presented.

AMS subject classifications: 65N06, 65N12, 65M06, 76M20

Key words: Tissue deformations, immersed interface method, linear stability analysis, finite difference method.

1 Introduction

In this paper, we consider a mathematical model developed in [3,8,9] for modeling deformations of contractile mesenchymal tissues. The tissues are considered to be composed of two inter-penetrating material phases: an aqueous phase and a cell-fiber phase. The aqueous phase is composed of all the water and dissolved extracellular components of the tissues. The cell-fiber phase consists of the cells and the remaining, generally fibrous, extracellular components. It is assumed that: (1) the two phases occupy complementary portions of the space, (2) the aqueous phase behaves as a Stokes fluid, (3) the stresses in the cell-fiber phase are dissipated by permanent deformation on the relevant time scale and can also be treated as a Stokes flow. These assumptions lead to the following system

*Corresponding author. *Email addresses:* qjiang2@gmail.com (Q. Jiang), zhilin@math.ncsu.edu (Z. Li), lubkin@math.ncsu.edu (S. R. Lubkin)

of partial differential equations (in 1D):

$$\frac{\partial\theta}{\partial t}+\frac{\partial(\theta v)}{\partial x}=0,\quad 0<x<L, \tag{1.1}$$

$$\frac{\partial}{\partial x}\left(\frac{1-\theta}{\varphi\theta}\cdot\frac{\partial p}{\partial x}-v\right)=0, \tag{1.2}$$

$$\frac{\partial}{\partial x}\left(2M\frac{\partial v}{\partial x}-p+\theta\psi+\sigma\ln(1-\theta)\right)=0, \tag{1.3}$$

where $0<\theta<1$ is the volume fraction of cells and fibers, v is the velocity of the cell-fiber phase, p is the pressure, φ is the drag coefficient, ψ is the contractility coefficient, σ is the swelling coefficient, and M is the viscosity coefficient of the cell-fiber fraction. Note that the parameters φ, M, ψ and σ are nonnegative and can depend on time, space, and θ.

A reasonable range of dimensional and non-dimensional parameters are presented in Table 1; see [3,8] for the references.

Table 1: Expected ranges of parameter values in which ϵ is a small positive number.

parameter	symbol	units	range
specific drag coefficient	φ	kg/m^3-sec	$10^{12}\sim10^{14}$
tissue viscosity	M	kg/m-sec	10^5
specific contractility coefficient	ψ	kg/m-sec^2	$10^3\sim10^4$
swelling number	σ	kg/m-sec^2	$10\sim10^3$
volume fraction of cell-fiber phase	θ_0	-	$\epsilon\sim(1-\epsilon)$

The boundary conditions (BC) are given as follows

$$v(0,t)=v(L,t)=0,\ \ \frac{\partial\theta}{\partial x}(0,t)=\frac{\partial\theta}{\partial x}(L,t)=0,\ \ \frac{\partial p}{\partial x}(0,t)=\frac{\partial p}{\partial x}(L,t)=0. \tag{1.4}$$

One way to model two adjacent tissues is to simply include them in the same equations and account for their different densities with θ. Thus a simple interaction between two tissues can be modeled with piecewise constant initial condition (see Fig. 1),

$$\theta(x,0)=\begin{cases}\theta_l, & \text{if } 0\le x<x_1 \text{ or } x_2<x\le L,\\ \theta_u, & \text{if } x_1\le x\le x_2,\end{cases} \tag{1.5}$$

where we use θ_l for the smaller constant (lower), and θ_u for the larger constant (upper).

In this paper, we will focus on simulating tissue deformations numerically for the one dimensional model. Note that the mathematical model is a non-linear, mixed (hyperbolic and elliptic) system of differential equations. Shock waves will be developed in