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A High Order Method for Determining the Edges in the Gradient of a Function

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Abstract. Detection of edges in piecewise smooth functions is important in many applications. Higher order reconstruction algorithms in image processing and post processing of numerical solutions to partial differential equations require the identification of smooth domains, creating the need for algorithms that will accurately identify discontinuities in a given function as well as those in its gradient. This work expands the use of the polynomial annihilation edge detector, (Archibald, Gelb and Yoon, 2005), to locate discontinuities in the gradient given irregularly sampled point values of a continuous function. The idea is to preprocess the given data by calculating the derivative, and then to use the polynomial annihilation edge detector to locate the jumps in the derivative. We compare our results to other recently developed methods.

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1 Introduction

Edge detection is fundamentally important in image reconstruction, feature extraction and several other applications. While many edge detection algorithms are available, [2, 5, 8, 10, 11, 13, 14], less attention has traditionally been paid to determining edges in the gradients of functions. However, such information can be very useful. For instance, solutions to partial differential equations that arise in gas dynamics and acoustic problems in heterogeneous media often have derivative discontinuities. Locating them can help high order post-processing of such numerical solutions. Their locations may also

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aid in determining domain decompositions that avoid shocks and contact discontinuities. This paper addresses the problem of determining discontinuities in the gradients of functions.

Most edge detection methods are either search based, [5,14], or zero-crossing based, [8]. Both types look directly for jump discontinuities. In contrast, the polynomial annihilation edge detector described in [2] determines intervals of smoothness from which the location of discontinuities in the function can be accurately identified. The method has several advantages over already existing methods, the most important being that it is applicable to multi-dimensional scattered data. Due to its variable order construction, it captures jumps located as close as one pixel apart as well as distinguishes them from steep gradients. The method is robust and is simple to implement numerically. Finally, it has limited dependence on outside thresholding present in many edge detectors.

An attempt to extend the polynomial annihilation edge detector to detect derivative discontinuities was made in [3]. However, the extension requires the use of several stencils, making it less efficient and robust compared to the original method in [2]. Furthermore, the method so far can be implemented in multi-dimensions only by using a dimension by dimension approach. This motivates us to develop a gradient edge detection method that can also be implemented on scattered data in higher dimensions.

The technique proposed in this paper detects jumps in the derivative of the given data using a two-pass approach. We first preprocess the data by numerically approximating the derivative. Then we use the polynomial annihilation edge detector to locate discontinuities in the approximate derivative. Our technique offers significant advantages over existing methods in that it is applicable to data on scattered grids and is multidimensional by design.

This paper is organized as follows: Section 2 reviews the polynomial annihilation edge detector and discusses its extension to capturing the derivative jump discontinuities. Section 3 explains our preprocessing algorithm in one and two dimensions followed by the edge detection procedure. Concluding remarks are given in Section 4.

2 Polynomial annihilation edge detection

2.1 Locating jump discontinuities in one dimension

The polynomial annihilation edge detector in [2] differs from other commonly used edge detectors chiefly in the fact that it looks for intervals of smoothness rather than looking for jump locations, [5,14]. In smooth intervals, the method annihilates the first m terms of the function's Taylor expansion. In non-smooth regions, the method essentially estimates the finite projection of the derivative – thereby locating the jump discontinuity.

To see how the edge detector works, let us consider a piecewise continuous function $f:[a,b]\rightarrow R$ known only on the set of discrete points $S=\{x_1,x_2,\cdots,x_N\}\subset[a,b]$. Assume that f has well defined one sided limits, $f(x\pm)$, at any point x in the domain. We denote by J the set of the points of discontinuity of f, that is, $J=\{\xi:a<\xi<b\}$, where ξ is a point