

Acoustic Scattering Cross Sections of Smart Obstacles: A Case Study

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Abstract. Acoustic scattering cross sections of smart furtive obstacles are studied and discussed. A smart furtive obstacle is an obstacle that, when hit by an incoming field, avoids detection through the use of a pressure current acting on its boundary. A highly parallelizable algorithm for computing the acoustic scattering cross section of smart obstacles is developed. As a case study, this algorithm is applied to the (acoustic) scattering cross section of a "smart" (furtive) simplified version of the NASA space shuttle when hit by incoming time-harmonic plane waves, the wavelengths of which are small compared to the characteristic dimensions of the shuttle. The solution to this numerically challenging scattering problem requires the solution of systems of linear equations with many unknowns and equations. Due to the sparsity of these systems of equations, they can be stored and solved using affordable computing resources. A cross section analysis of the simplified NASA space shuttle highlights three findings: i) the smart furtive obstacle reduces the magnitude of its cross section compared to the cross section of a corresponding "passive" obstacle; ii) several wave propagation directions fail to satisfactorily respond to the smart strategy of the obstacle; iii) satisfactory furtive effects along all directions may only be obtained by using a pressure current of considerable magnitude. Numerical experiments and virtual reality applications can be found at the website: <http://www.ceri.uniroma1.it/ceri/zirilli/w7>.

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1 Introduction

A smart obstacle is an obstacle that, when hit by an incoming acoustic wave, responds by circulating a pressure current on its boundary according to the goals of the object's designed detection strategy. The pressure current is a quantity with physical dimensions of pressure divided by time. The most general class of smart obstacles considered by the authors (ghost obstacles) are obstacles that, when hit by an incoming wave, attempt to produce a scattered acoustic field that would be expected from a virtual obstacle (ghost) present under the same circumstances. The virtual object is designed to differ from the smart obstacle in both shape and position in space [5, 11]. This general class of smart obstacles includes, as special cases, furtive obstacles and masked obstacles. Furtive obstacles try to avoid detection when hit by an incoming wave by scattering a small amplitude wave [13]. When hit by an incoming wave, masked obstacles try to avoid detection by scattering an acoustic field that would be produced under the same circumstances by a virtual obstacle (a mask) that is different from the masked object in shape [9, 10]. Passive obstacles are obstacles that, when hit by an incoming acoustic field, do not respond by circulating a pressure current on their boundary as a way of manipulating their scattered field. Models of "smart obstacles" and scattering phenomena, introduced in [5, 9–11, 13, 15], are expressed as optimization (or optimal control) problems and partial differential equations. Some attempts at solving the time-harmonic inverse acoustic scattering problems associated with the inversion of these models by smart obstacles are described in [7, 8]. The mathematical model for the time-dependent scattering problem stated above, which is a generalization of the model considered here, is an optimal control problem for the wave equation [9, 11, 13]. Recently, physical examples of smart objects, such as phase-switched screens, have been designed and built (see, for example, [1–4]). Phase-switched screens are used to build radar absorbers and are an example in the electromagnetic domain of a smart object that pursues the goal of being furtive. In this paper, we develop a parallel numerical method for computing the acoustic scattering cross section of realistic smart obstacles. We use this algorithm to study the acoustic scattering cross section of smart furtive obstacles and compare the cross section with the scattering cross section of passive obstacles, that is, we study the furtivity effect. Cross sections of realistic smart obstacles at the wavelengths considered here have not been studied previously using mathematical models based on partial differential equations that describe the full wave propagation phenomenon. The behavior of these cross sections is relevant to understanding the impact of the smart strategies (i.e., the pressure currents used) on the fields scattered, and to the study of the relation between the geometry of the obstacles and the properties of the corresponding cross sections. Let \mathbb{R}^3 be three-dimensional real Euclidean space. To compute the cross section of a smart obstacle, we must solve the corresponding smart furtive obstacle scattering problem several times, which can be stated as follows: *given an incoming time-harmonic acoustic field propagating in \mathbb{R}^3 and a bounded obstacle $\Omega \subset \mathbb{R}^3$, which is non-empty and characterized by a nonnegative acoustic boundary impedance χ , find a time-harmonic pressure current circulating on the boundary of Ω*