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An Accurate Numerical Scheme for Maxwell Equation with CIP-Method of Characteristics

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Abstract. A new multi-dimensional scheme for the Maxwell equations is established by the CIP method in combination with the method of characteristics (CIP-MOC). In addition, the CIP-MOC can be extended to arbitrary grid system by the Soroban grid without losing the third-order accuracy. With the accuracy fixed, the grid points required for the CIP are 40 times less than the conventional schemes like the FDTD in three dimensions. Numerical solutions obtained by the CIP-MOC are compared with analytical solution and the FDTD in plane-wave scattering by a perfectly-conducting circular cylinder, and the CIP-MOC agrees very well with analytical solutions. The Soroban grid is also applied to the Vlasov equation that describes the kinematics of plasmas that is frequently combined with the Maxwell equation. The adaptively moving points in velocity space are similar to the particle codes but can provide accurate solutions.

Key words: Maxwell equation; Vlasov equation; CIP method; method of characteristics; Soroban grid.

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1 Introduction

Although the research of electromagnetic waves has a long history, there is still growing interest in its numerical solution under complex shape boundary as well as material properties of structures and so on. At the frontier of such simulation, we need a reliable numerical schemes that can provide the correct prediction even for a new regime not explored before. Owing to unknown reliability of the numerical solutions, the details of many unknown data are estimated on the basis of some experimental measurements.

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However, it is obvious that such empirical law obtained by measurements is not flexible and it requires a lot of efforts and time. Therefore, in order to effectively cope with all kinds of complex and usual phenomena, numerical analysis has been attracting attention in order to overcome these problems. Since performance of computers has been increasing these days, this option will be very promising for electromagnetic field analysis once a reliable scheme is established.

A number of numerical schemes have been developed from a continuum version of the Maxwell equations to a discretized version by using various differencing schemes in time and space. Some of the well-known schemes are the Finite-Difference Time-Domain (FDTD) method [1], the Finite Element Method (FEM) [2], and so forth. The ray-tracing method can be applied to refraction and reflection on the material surface, however, when there are some complicated structures in a domain, the number of lights that have to be traced becomes huge.

The FDTD method has been commonly used because it is very easy to make the program and treat current sources, dielectrics and conductors consistently. Since this scheme has already been established for a long time, it has been employed for all kinds of commercial software.

However, the FDTD method is the second order in time and space and its phase error is quite large in short wavelength component. Since electromagnetic waves are composed of arbitral wavelengths or frequencies and the numerical resolution of wave motions is determined from the minimum wave number within the frequency spectrum, the required number of grid should be enormous in order to provide adequate numerical resolution beyond the Rayleigh into the resonance.

Therefore, the application of the CIP method [3–6] to the analysis of computational electromagnetics seems to be promising because of its non-dispersive characteristics. Since the CIP method can accurately solve the hyperbolic equations and the Maxwell equations are also a system of hyperbolic partial differential equations (PDEs) , the CIP method can be similarly applied to the propagation of characteristics that appears in electromagnetic waves.

The objective of this study is to explore the benefits of using the theory of characteristics in developping accurate and efficient numerical algorithms for computational electromagnetics. The present work adopts the CIP method in combination with the numerical method of characteristics (MOC). Hereafter, it is called the CIP-MOC method. We have already applied the CIP-MOC method to the multi-dimensional shallow water equations [7] with directional splitting, and the same procedure can be applied to the Maxwell equations as well.

Although the CIP is the third-order in time and space in uniform grid, the accuracy of any schemes including the CIP becomes the first-order in time and space for abruptly changing the size of meshes once the metric tensors for coordinate transformation are estimated by finite difference methods. In order to resolve this problem, a new body-fitted grid system, the Soroban-grid was proposed in the previous paper [8]. One of the advantages of the Soroban grid is that it can keep the third-order accuracy in time and