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# Complex Transition of Double-Diffusive Convection in a Rectangular Enclosure with Height-to-Length Ratio Equal to 4: Part I

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**Abstract.** This is the first part of direct numerical simulation (DNS) of double-diffusive convection in a slim rectangular enclosure with horizontal temperature and concentration gradients. We consider the case with the thermal Rayleigh number of $10^5$, the Pradtle number of 1, the Lewis number of 2, the buoyancy ratio of composition to temperature being in the range of [0,1], and height-to-width aspect ration of 4. A new 7th-order upwind compact scheme was developed for approximation of convective terms, and a three-stage third-order Runge-Kutta method was employed for time advancement. Our DNS suggests that with the buoyancy ratio increasing form 0 to 1, the flow of transition is a complex series changing from the steady to periodic, chaotic, periodic, quasi-periodic, and finally back to periodic. There are two types of periodic flow, one is simple periodic flow with single fundamental frequency (FF), and another is complex periodic flow with multiple FFs. This process is illustrated by using time-velocity histories, Fourier frequency spectrum analysis and the phase-space trajectories.

**AMS subject classifications**: 65Y20, 34C28, 70K50

**Key words**: Double diffusive convection, transition, periodic motion, chaotic motion, high order compact.

## 1 Introduction

Double-diffusive convection motion, as a common flow phenomena in nature, has been studied for a long time. It exists in many procedures containing multi-physics and multi-process coupling and interaction. For instance, the global ocean circulation driven by

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interaction of temperature and salinity, diffusion of pollution and temperature in soil contamination and atmospheric pollution, solar energy concentration, nuclear reactor cooling. Furthermore, the multi-diffusive convection exists in some special fields, such as the action of crystal growth and solidification of metallic alloys.

At present, cavity physics modeling [1–3], Boussinesq modeling [4,5] and $k$-$\varepsilon$ modeling [6–8] are extensively used in investigation of the double-diffusive convection problem. It is mainly influenced by parameter set $\Omega$ ($Ra_T$, $N$, $Pr$, $Le$, $A$) [9], where $Ra_T$ is the thermal Rayleigh number, $N$ is the buoyancy ratio of composition to temperature, $Pr$ is the Prandtl number, $Le$ is the Lewis number (the ratio of composition to heat diffusivity) and $A$ is the height-to-width aspect ratio. It is far too difficult to have a systematic investigation to cover all of the parameter set. For decades, researchers have investigated double-diffusive convection by choosing different parameter separately. Quon [9, 10], Lee [4,5] and Tsitverblit [13] investigated the multi-cell flow structure and multi-stratified stable region with various values of $A$, $Le$, $N$ and different boundary conditions. Quon [9] investigated the mechanism of spontaneous, abrupt changes in thermohaline circulation in an idealized context, by using a two-dimensional Boussinesq fluid in rectangular containers with Rayleigh number up to $10^5$.

The double-diffusive convection is a strong nonlinear coupled process, which contains the steady flow, the periodic flow, the quasi-periodic flow, the chaos, and even the turbulence. A similar problem, i.e., a lid-driven square cavity flow, has already been investigated for decades. Garcia [12] has proposed a comprehensive long term dynamic behavior of such problem. He observed that for low Reynolds numbers, the solution was stationary. For moderate Reynolds numbers, it was time periodic; and for high Reynolds numbers, the solution was neither stationary nor time periodic: the solution becomes chaotic. Some relevant papers about this topic can also be found in [12]. Tsitverblit and Kit [13] reported that a vertical rectangular enclosure was characterized by complex steady bifurcation phenomena, containing stably stratified brine and differentially heated from its side walls. Nishimura [11] has studied the Hopf bifurcation with various values of $N$ by using the Galerkin finite element method. He claimed that the key mechanism for the oscillatory flow was that the unstable stratified region of species shifts from the central part of the enclosure to the upper and lower parts and vice versa in a time-periodic sense, due to the interaction of heat and mass transfer with different diffusivities near the vertical walls. Recently, Papanicolaou and Belessiotis [14] reported that the unsteadiness in the laminar-flow regime is due to interactions between the main flow cells affecting the bulk of the enclosure, whereas in the turbulent flow regime due to thermal release and motion along the bottom surface through studying the double-diffusive natural convection in an asymmetric trapezoidal enclosure. Masuda et al. [15] reported three types of peculiar oscillating convection in porous medium, called chaotic oscillations, sudden steady state and re-oscillation in their paper.

The DNS of double-diffusive convection problem has attracted considerable attentions. The rapid development of computing techniques provided powerful capacity to anatomize such complex problems, especially the complicate transition [9, 11, 14, 15] in