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Computation of Shock Profiles in Radiative Hydrodynamics

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Abstract. This article is devoted to the construction of a numerical scheme to solve the equations of radiative hydrodynamics. We use this numerical procedure to compute shock profiles and illustrate some earlier theoretical results about their smoothness and monotonicity properties. We first consider a scalar toy model, then we extend our analysis to a more realistic system for the radiative hydrodynamics that couples the Euler equations and an elliptic equation.

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1 Introduction

The aim of this paper is to construct a numerical scheme to compute shock profiles for some models of radiative hydrodynamics. Such models couple a hydrodynamics part, typically the Euler equations of compressible gas dynamics, with a nonlocal source term. In what follows, the nonlocal operator will be a convolution operator on the real line. Our numerical procedure is based on a splitting strategy. In a first time substep, we solve the hydrodynamics part by means of a conservative low-diffusive scheme (either the Godunov scheme or the so-called Lagrange-projection scheme). In a second time substep, we solve the radiative part of the equations. This amounts to solving a differential equation where the source term is an integral operator. In the models we are interested in, the

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integral operator stems from the resolution of an elliptic equation. There are two possible approaches to discretize the radiative part: we can either try to solve the elliptic equation, or use a quadrature formula for the integral. We shall compare both methods.

Our main motivation is the computation of shock profiles and the checking of some theoretical results that were derived in [9,18] for a scalar model, and in [14,15] for a more complete model. Because shock profiles are asymptotically constant at $\pm\infty$, we shall see that it is reasonable to truncate the elliptic equation on the bounded domain where the solution is computed by forcing Dirichlet boundary conditions. If one is interested in solving the equations in a general situation, namely for a very general class of initial data, the nonlocal operator will raise some additional difficulties because it precludes the finite speed of propagation. Therefore, the choice of the computational domain will play a major role. Another motivation for this work is the behavior of large amplitude shock profiles. This is well understood for the scalar model studied in [9, 18], where it was shown that shock profiles of arbitrary strength are monotone functions. For the more complete model studied in [14,15], there are no theoretical results and we want to check that shock profiles for this model capture the main features of radiative shocks, see, e.g., [16,21]. In particular, we shall check numerically that large shock profiles exhibit a post-shock region where the temperature is non-monotone. This property was already displayed for other models in radiative hydrodynamics, see, e.g., [2,3,16,21]. The main point is that our method does not require any mesh refinement near the shock to exhibit these peaks in the temperature profile.

The paper is divided as follows. In Section 2, we present the scalar toy model for radiating gases and develop two numerical methods. Our first method is based on the discretization of an elliptic equation, while the second method is based on a quadrature rule for the integral representation of the solution to this elliptic equation. In Section 3, we introduce a more involved model that couples the Euler equations and an elliptic equation. We develop some numerical methods for this system that have some similarities with the methods introduced for the toy model. We illustrate both sections with some numerical experiments that confirm some earlier theoretical results.

2 Numerical schemes for a toy model

2.1 The numerical schemes

In this section, we propose a numerical procedure to solve the toy model:

$$\begin{cases} \partial_t u + \partial_x (u^2/2) = q - u, & t \geq 0, \ x \in \mathbb{R}, \\ -\partial_{xx} q = u - q. \end{cases} \tag{2.1}$$

Observe that the elliptic equation for q can be solved explicitly using a convolution:

$$q(t,x) = \frac{1}{2} \int_{\mathbb{R}} \mathrm{e}^{-|x-y|} u(t,y) \, dy. \tag{2.2}$$