## **Emergence of Community Structure in the Adaptive Social Networks**

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Received 18 October 2009; Accepted (in revised version) 16 December 2009

Communicated by Dietrich Stauffer

Available online 17 May 2010

**Abstract.** In this paper, we propose a simple model of opinion dynamics to construct social networks, based on the algorithm of link rewiring of local attachment (RLA) and global attachment (RGA). Generality, the system does reach a steady state where all individuals' opinion and the complex network structure are fixed. The RGA enhances the ability of consensus of opinion formation. Furthermore, by tuning a model parameter *p*, which governs the proportion of RLA and RGA, we find the formation of hierarchical structure in the social networks for *p* > *p*<sub>c</sub>. Here, *p*<sub>c</sub> is related to the complex network size *N* and the minimal coordination number 2*K*. The model also reproduces many features of large social networks, including the "weak links" property.

PACS: 87.23.Ge, 89.75.Fb

Key words: Opinion dynamics, social network, community structure, weak links property.

## 1 Introduction

Networks have in recent years undergone a remarkable development and have emerged as an invaluable tool for describing and quantifying complex systems in many branches of science [1]. It has been realized that many real complex networks, including social networks such as peer-to-peer social networks [2] and acquaintance networks [3], the technology networks such as the power grids [3,4], and biological networks such as the food webs [5] and metabolic networks [6], all share some distinctive characteristic properties. One such property is the "small-world effect", which means that the average shortest path length between vertices in network is short, usually scaling logarithmically with the size N of network. Another is the hierarchical structure, which means that vertices

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are divided into groups that further subdivided into groups of groups, and so forth over multiple scales, in network [1]. Examples include the ecological niches in food webs, the modules in biochemical networks and communities in social networks [1,7–9].

The community structure, the gathering of vertices into groups such that there is a higher density of edges within groups than between them [9], is a crucial property of social network and draws many scientists' attention to research how to detect and identify the communities within networks. Recently, a number of approaches, such as the divisive algorithm based on the edge betweenness [8, 10], the modularity algorithm [11], the self-contained algorithm [12] and a physical approach based on notions of voltage drops across networks [13], have been proposed. Another important problem is the mechanism of emergence of the hierarchical structure, especially the communities in social networks. Clauset et al. proposed a hierarchical random graph to study the hierarchical structure and predict the missing links in networks [1]. González et al. proposed a model of mobile agents to construct social networks and found the emergence of a giant cluster in the universality class of two-dimensional percolation above a critical collision rate [14]. And Kumpula et al. proposed a weight-topology dynamics model to generate a weighted networks with communities [15]. Nevertheless, the understanding of the hierarchical structure property in complex networks remains a challenge.

On the other hand, recent years have witnessed an attempt by physicists to research the collective phenomena emerging from the interactions of individuals as elementary units in social structure [16]. Castellano et al. review a wide list of topics of collective phenomena ranging from opinion and cultural and language dynamics to crowd behavior, hierarchy formation, human dynamics, and social spreading [16]. Many previous works have studied on static substrates: the interaction pattern is fixed and only opinion, not connections, are allowed to change [16]. This is the case of the dynamics on networks. The opposite case is the dynamics of networks: the links between vertices are formed or removed according to such fixed vertex properties, such as the network formation depending on the present degree [17] and weights [18] of the existing vertices. In fact, real social systems are mostly in between these two extreme cases: both intrinsic property of vertices (like opinions) and connections among them vary in time over comparable temporal scales. The interaction between those two evolutions is then a natural issue to be researched. Holme and Newman [19] proposed a simple model for the coevolution of opinions and social networks in a situation in which both adapt to the other with a single parameter  $\phi$ . They found that the model undergoes a continuous phase transition as the parameter  $\phi$  is varied. Stauffer et al. proposed the model of the coevolution of individual economic characteristics and socioeconomic networks, where links between agents with similar characteristics are more stable than those between agents with vastly different characteristics [20]. They found that a simple scaling law describes the number of distinct surviving characteristic realizations as a function of the number of agents and the number of possible distinct characteristics realizations. Allahverdyan and Petrosyan studied a model for a statistical network formed by interactions between its nodes and links, where each node can be in one of two states and the node-link interaction facilitates