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Finite Volume Evolution Galerkin Methods for the Shallow Water Equations with Dry Beds

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Abstract. We present a new Finite Volume Evolution Galerkin (FVEG) scheme for the solution of the shallow water equations (SWE) with the bottom topography as a source term. Our new scheme will be based on the FVEG methods presented in (Noelle and Kraft, J. Comp. Phys., 221 (2007)), but adds the possibility to handle dry boundaries. The most important aspect is to preserve the positivity of the water height. We present a general approach to ensure this for arbitrary finite volume schemes. The main idea is to limit the outgoing fluxes of a cell whenever they would create negative water height. Physically, this corresponds to the absence of fluxes in the presence of vacuum. Well-balancing is then re-established by splitting gravitational and gravity driven parts of the flux. Moreover, a new entropy fix is introduced that improves the reproduction of sonic rarefaction waves.

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1 Introduction

The shallow water equations (SWE) are a mathematical model for the movement of water under the action of gravity. Mathematically spoken, they form a set of hyperbolic conservation laws, which can be extended by source terms like the influence of the bottom topography, friction or wind forces. In this case, we will speak of a balance law. For simplicity, this work will consider the variation of the bottom as the only source term.

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Many important properties of the model rely on the fact that the water height is strictly positive. Despite this, typical relevant problems include the occurrence of dry areas, like dam break problems or the run-up of waves at a coast, with tsunamis as the most impressive example. So for simulations of these problems, we have to develop numerical schemes that can handle the (possibly moving) shoreline in a stable and efficient way. Another crucial point in solving balance laws is the treatment of the source terms. For precise solutions, it is necessary to evaluate the source term in such a way that certain steady states are kept numerically, i.e., the numerical flux and the numerical source term cancel each other exactly for equilibrium solutions.

In the last years, many groups contributed to the solution of the difficulties described above. In [2], Audusse et al. proposed a reconstruction procedure where the free surface and water height are reconstructed and the bottom slopes are computed from these. This guarantees the positivity of the water height and gives a well-balanced scheme at the same time. Begnudelli and Sanders developed a scheme for triangular meshes including scalar transports in [3]. They proposed a strategy how to exactly represent the free surface in partially wetted cells, leading to improved results at the wetting/drying front. In [8], Brufau et al. analyze how to deal with flow on an adverse slope. They locally modify the bottom topography in certain situations to avoid unphysical run-ups or wave creation at the dry boundary. Gallardo et al. discussed various solutions of the Riemann problem at the front and used them in a modified Roe scheme. They then used the local hyperbolic harmonic method from Marquina (cf. [24]) in the reconstruction step to achieve higher order, see [9]. Kurganov and Petrova proposed a central-upwind scheme that is well-balanced and positivity preserving in [13]. It is based on a continuous, piecewise linear approximation of the bottom topography and performs the computation in terms of the free surface instead of the relative water height to simplify the well-balancing. The last feature is also a building block in the work of Liang and Marche [16]. They also provide a method to extend this well-balancing feature to situations including wetting/drying fronts. Liang and Borthwick [15] used adaptive quad-tree grids to improve the efficiency of their schemes. Wetting and drying effects are handled as well as friction terms. In the context of residual distribution methods, Ricchiuto and Bollermann developed a positivity preserving and well-balanced scheme for unstructured triangulations [26].

The finite volume evolution Galerkin (FVEG) methods developed by Lukáčová, Morton and Warnecke, cf. [18–20], have been successfully applied to the SWE in [19]. They are based on the evaluation of so called *evolution operators* which predict values for the finite volume update. Thanks to these operators, the schemes take into account all directions of wave propagation, enabling them to precisely catch multidimensional effects even on Cartesian grids. These schemes show a very good accuracy even on relatively coarse meshes compared to other state of the art schemes and they are also competitive in terms of efficiency (cf. [19]).

However, the existing FVEG schemes are not able to deal with dry boundaries. Thus in this work we will present a method to preserve the positivity of the water height with an arbitrary finite volume method. To achieve this, we reduce the outflow on draining