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A Normal Mode Stability Analysis of Numerical Interface Conditions for Fluid/Structure Interaction

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Abstract. In multi physics computations where a compressible fluid is coupled with a linearly elastic solid, it is standard to enforce continuity of the normal velocities and of the normal stresses at the interface between the fluid and the solid. In a numerical scheme, there are many ways that velocity- and stress-continuity can be enforced in the discrete approximation. This paper performs a normal mode stability analysis of the linearized problem to investigate the stability of different numerical interface conditions for a model problem approximated by upwind type finite difference schemes. The analysis shows that depending on the ratio of densities between the solid and the fluid, some numerical interface conditions are stable up to the maximal CFL-limit, while other numerical interface conditions suffer from a severe reduction of the stable CFL-limit. The paper also presents a new interface condition, obtained as a simplified characteristic boundary condition, that is proved to not suffer from any reduction of the stable CFL-limit. Numerical experiments in one space dimension show that the new interface condition is stable also for computations with the non-linear Euler equations of compressible fluid flow coupled with a linearly elastic solid.

AMS subject classifications: 65M12, 35L60, 35L65

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1 Introduction

The subject of this article is the stability of numerical approximations of fluid-structure interaction problems with particular emphasis on high-speed flow applications. This

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type of fluid-structure interaction occurs in many application areas including aeroelasticity, modeling of explosives, and others. Numerous previous works have developed numerical methods for this type of problem (for example [1,2]).

Although many large scale computations of fluid-structure interaction have been performed to date, see, e.g., [3, 4], the theory of stability and convergence of numerical schemes for such problems is less developed. The aim of this paper is to improve the understanding of the underlying mathematical properties of approximations for fluid-structure interaction problems, and thereby facilitate improvements in accuracy and efficiency of the numerical methods.

We introduce a one dimensional linear problem, obtained by linearizing the equations of inviscid compressible fluid flow coupled with a linear elasticity model. This simple model, which consists of two coupled wave equations, gives insight into the different numerical behaviors that occur at the interface between the materials. It turns out that for explicit time stepping schemes, one critical parameter affecting the numerical stability is the ratio of the densities on either side of the interface. For problems in aeroelasticity, the density of the solid material is usually several orders of magnitude larger than the density of the fluid. However, in other fluid-structure interaction applications, such as explosives modeling, the density ratio can be very close to one. Although standard numerical methods often can be tuned to a specific density ratio, a coupling method that performs well regardless of the density ratio between the solid and fluid would be more convenient. This is especially true for non-linear problems where the density ratio can vary widely from point to point along the interface and also in time. One remedy for instabilities is to switch to implicit coupling. However, when the objective is to resolve all time scales (as if often the case in high-speed compressible applications), explicit methods are preferable because of their lower computational cost. This paper shows that by choosing appropriate numerical interface conditions it is possible to have a stable explicit method for any density ratio, that does not suffer from any CFL restriction other than that of the numerical schemes in the respective solid and fluid parts of the domain.

Section 2 introduces a one dimensional fluid-structure example problem, where a few standard methods with explicit time stepping are shown to be stable or unstable depending on the density ratio. A simplified model problem is then derived via linearization. The explanation for the unstable behavior is captured in the analysis of the model problem presented in Section 3. Section 3 also introduces a new numerical interface condition that is proved to be stable with explicit time stepping for any density ratio. Numerical experiments with the model problem in Section 4 compare the performance of the different interface conditions, confirming the analytical results. Finally, in Section 4.1, the generalization of the new stable interface condition to the fluid/structure example from Section 2, shows stable behavior in numerical experiments with both small and large density ratios.