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On Approximating Strongly Dispersion-Managed Solitons

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Abstract. We use a generalized scaling invariance of the dispersion-managed nonlinear Schrödinger equation to derive an approximate function for strongly dispersion-managed solitons. We then analyze the regime in which the approximation is valid. Finally, we present a method for extracting the underlying soliton part from a noisy pulse, using the resulting approximate formula.

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Key words: Lasers, fiber optics, soliton, dispersion-management, scaling invariance, Gaussian ansatz.

1 Introduction

The technique of dispersion-management (DM), developed to improve the performance of the fiber-optic transmission lines in 1990s, has become an essential component of modern optical fiber communication systems [1, 2]. Technically speaking, DM is realized by concatenating fiber sections with different chromatic dispersion to build the transmission line. Mathematically, dispersion-managed systems are described by the nonlinear Schrödinger equation (NLS) equation with periodically varying dispersion (see, e.g., [3,4]):

$$i\frac{\partial u}{\partial t}+\frac{1}{2}D(t/t_a)\frac{\partial^2 u}{\partial x^2}+|u|^2u=0, \tag{1.1}$$

where all quantities are expressed in dimensionless units, and where, t stands for the propagation distance and x stands for time. The function $D(t/t_a)$ represents the local value of fiber dispersion. The quantity t_a appearing in Eq. (1.1) is the characteristic (dimensionless) distance between amplifiers, which we assume to be small compared to the nonlinear distance and the dispersion distance; that is, $t_a \ll 1$. For example, with a typical

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amplifier spacing of about 50 km and typical nonlinear distance of about 400 ∼ 1000 km, it is $t_a = 0.05 \sim 0.125$. Eq. (1.1) contains both large and rapidly varying terms and thus is not useful for studying the long term behavior of solutions. By employing appropriate multiple-scale expansions on Eq. (1.1), one obtains an integro-differential equation, called dispersion-managed nonlinear Schrödinger equation (DMNLS) equation, governing the long-term dynamics of such systems [4, 5]:

$$i\frac{\partial u}{\partial t}+\frac{1}{2}\bar{d}\frac{\partial^2 u}{\partial x^2}+\iint u_{(x+x')}u_{(x+x'')}u^*_{(x+x'+x'')}R_{(x',x'')}\,dx'dx''=0, \tag{1.2}$$

where $\bar{d}$ is the average dispersion, and where the integral kernels $R(x',x'')$ and $r(y)$ are respectively,

$$R(x',x'')=\mathrm{ci}(x'x''/s)/(2\pi|s|), \quad r(y)=\sin(sy)/(4\pi^2 sy).$$

Here the parameter s, called the reduced mapstrength, is defined by

$$s=\frac{1}{4}\int_0^1 |\Delta D(\zeta)|d\zeta,$$

where $\Delta D(\cdot)$ is the zero-mean variation [4] in $d(\cdot)$:

$$d(t/t_a)=\bar{d}+\Delta D(t/t_a).$$

By performing an appropriate nondimensionalization, the average dispersion can be normalized to be $\bar{d}=1$ in the abnormal regime [14]. In what follows, we will assume this has been done. Moreover, Eq. (1.2) reduces to the standard NLS equation when $s=0$. The DMNLS equation has been extensively studied in the literature of fiber optics [6–14]. More interestingly, certain types of mode-locked lasers are also dispersion-managed [15], where Eq. (1.1) is again the appropriate model for the pulse dynamics [16, 17]. And it has been suggested recently that Eq. (1.2) describes the asymptotic behavior of the pulses in these mode-locked laser systems as well. Because of the limitation of space, a lot of physical details are left out here for both the fiber-optic communication systems and the lasers, and interested readers are encouraged to consult the cited works and the references therein. In many applications, the lasers are required to produce ultrashort pulses (e.g., fetosecond) [18], which are stable and soliton-like, and hence are usually referred to as the DM solitons (DMS). From a mathematical point of view, the DMS can be associated to either a solution of Eq. (1.1), which is localized in x and periodically varying in t, or a traveling-wave solution of the DMNLS equation (1.2), which preserves its shape during propagation [10].

Considerable efforts have been dedicated to approximations of the periodic solution of Eq. (1.1) [19, 21, 22]. On the other hand, it should also be beneficial to have an approximate function of the DMS as traveling-wave solutions of the DMNLS equation[†].

[†]To avoid confusion, we only refer to the traveling-wave solutions of Eq. (1.2) as the DMS hereafter.