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An Indirect-Forcing Immersed Boundary Method for Incompressible Viscous Flows with Interfaces on Irregular Domains

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Abstract. An indirect-forcing immersed boundary method for solving the incompressible Navier-Stokes equations involving the interfaces and irregular domains is developed. The rigid boundaries and interfaces are represented by a number of Lagrangian control points. Stationary rigid boundaries are embedded in the Cartesian grid and singular forces at the rigid boundaries are applied to impose the prescribed velocity conditions. The singular forces at the interfaces and the rigid boundaries are then distributed to the nearby Cartesian grid points using the immersed boundary method. In the present work, the singular forces at the rigid boundaries are computed implicitly by solving a small system of equations at each time step to ensure that the prescribed velocity condition at the rigid boundary is satisfied exactly. For deformable interfaces, the forces that the interface exerts on the fluid are computed from the configuration of the elastic interface and are applied to the fluid. The Navier-Stokes equations are discretized using finite difference method on a staggered uniform Cartesian grid by a second order accurate projection method. The ability of the method to simulate viscous flows with interfaces on irregular domains is demonstrated by applying to the rotational flow problem, the relaxation of an elastic membrane and flow in a constriction with an immersed elastic membrane.

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Key words: Incompressible Navier-Stokes equation, fast Poisson solvers, immersed boundary method, projection method, Cartesian grid, irregular domain, finite difference methods.

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1 Introduction

Flow problems involving the deformable interface and complex geometries still pose a difficult challenge in computational fluid dynamics. One of the challenges in these problems is that the fluid motion, the motion of the deformable interface and the interaction with the immersed rigid boundaries must be computed simultaneously. This is necessary in order to account for the complex interaction between the fluid, the interfaces and the immersed boundaries. Fig. 1 shows an illustration of such problems involving the rigid boundary and fixed/deformable interface embedded in a uniform Cartesian grid.

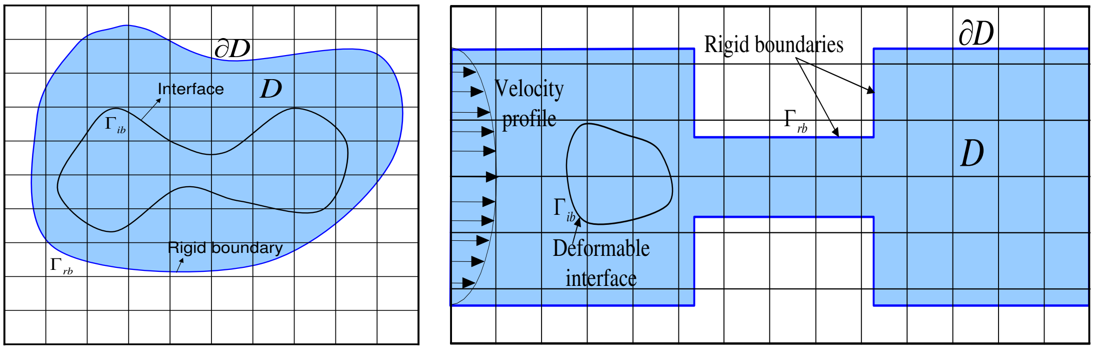

Figure 1: Two typical domains with the rigid boundary and fixed/deformable interface immersed in a uniform Cartesian grid.

Conventional methods for solving the Navier-Stokes equations with rigid immersed boundaries include the body-fitted or structured grid approach. In this approach, the Navier-Stokes equations are discretized on a curvilinear grid that conforms to the immersed boundary and hence the boundary conditions can be imposed easily. The disadvantage of this method is that robust grid generation is required to account for the complexity of the immersed boundaries. An alternative and popular approach for solving complex viscous flows involving the interfaces on a complex geometry is the Cartesian grid method which solves the governing equations on a Cartesian grid and has the advantages of retaining the simplicity of the Navier-Stokes equations on the Cartesian coordinates and enabling the use of fast solvers.

One of the most successful Cartesian grid methods is Peskin's immersed boundary method [25]. This method was originally developed to study the fluid dynamics of blood flow in the human heart [24]. The method was developed further and has been applied to many biological problems involving flexible boundaries [7,34]. The immersed boundary method has also been applied to handle problems with immersed boundaries [11,20]. In order to deal with rigid immersed boundaries, Lai and Peskin [11] proposed to evaluate the force density using an expression of the form,

$$\mathbf{f}(s,t)=K_r(\mathbf{X}^e(s)-\mathbf{X}(s,t)), \tag{1.1}$$

where K_r is a constant, $K_r \gg 1$, $\mathbf{X}$ and $\mathbf{X}^e$ are the arc-parametrization of the computed and