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The Approximate Solutions of FPK Equations in High Dimensions for Some Nonlinear Stochastic Dynamic Systems

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Abstract. The probabilistic solutions of the nonlinear stochastic dynamic (NSD) systems with polynomial type of nonlinearity are investigated with the subspace-EPC method. The space of the state variables of large-scale nonlinear stochastic dynamic system excited by white noises is separated into two subspaces. Both sides of the Fokker-Planck-Kolmogorov (FPK) equation corresponding to the NSD system is then integrated over one of the subspaces. The FPK equation for the joint probability density function of the state variables in another subspace is formulated. Therefore, the FPK equation in low dimensions is obtained from the original FPK equation in high dimensions and it makes the problem of obtaining the probabilistic solutions of large-scale NSD systems solvable with the exponential polynomial closure method. Examples about the NSD systems with polynomial type of nonlinearity are given to show the effectiveness of the subspace-EPC method in these cases.

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Key words: Nonlinear stochastic dynamic systems, large-scale systems, probability density function, Fokker-Planck-Kolmogorov equation, subspace.

1 Introduction

FPK equations have been widely used in statistical mechanics and other areas of science and engineering since it was formulated almost one century ago [1–4]. It is known that practical problems are frequently described as multi-degree-of-freedom (MDOF) or high-dimensional systems with random excitations. Therefore, the probabilistic solutions of

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nonlinear stochastic dynamic MDOF systems are needed in many areas of science and engineering. However, only in some limited cases, the exact probabilistic solutions of two-degree-of-freedom systems are obtainable [4, 5]. It is well known that even the solutions of nonlinear single-degree-of-freedom (SDOF) systems attracted much attention in the last decades [6–24]. The challenge in analyzing the nonlinear stochastic dynamic (NSD) systems lies in the difficulties in obtaining the probability density function (PDF) of the system responses. Even if the probabilistic solution of a NSD system is governed by FPK equation, it is still difficult to obtain the exact solution if the MDOF system is nonlinear. Therefore, some methods were proposed for the approximate solutions of FPK equations. The most frequently employed approximation method is the equivalent linearization (EQL) procedure [3, 6, 7]. The advantage of the EQL method is that it can be used for analyzing large-scale NSD systems, but it is considered unsuitable when the system is highly nonlinear or multiplicative random excitations are present, because in either case the probability distribution of the system response is usually far from being Gaussian. To improve the accuracy of the approximate solution, various approximation methods were proposed, such as non-Gaussian closure method with Gram-Charlier series or Hermite-polynomial approximation [8, 9], maximum entropy method [10, 11], stochastic average method [12, 13], perturbation method [14–16], equivalent nonlinear system method [17–19], finite element method [20, 21], finite difference method [21, 22], and exponential polynomial closure (EPC) method [23–25]. It is well known that all these methods are only suitable for analyzing the one, two or at most few-degree-of-freedom systems. Monte Carlo simulation (MCS) is versatile [26, 27], but the amount of computation with it is usually unacceptable for estimating the PDF solutions of the responses of large-scale NSD systems, especially for small probability problems. The numerical convergence and stability are also challenge problems for analyzing nonlinear stochastic dynamic systems with MCS. It is seen that the problem of obtaining the PDF solutions of large-scale NSD systems has been a challenge in this area for decades. There is no effective methods available for obtaining the acceptable approximate PDF solutions of large-scale NSD systems with high nonlinearity. Recently, a new method named subspace method was proposed for the approximate PDF solutions of large-scale nonlinear stochastic dynamic systems [28]. In this paper, the subspace method is applied to analyze the nonlinear stochastic dynamic systems with polynomial types of nonlinearities to further examine the effectiveness of the subspace method in these cases. With subspace method, the problem of solving the FPK equation in high-dimensions is reduced to the problem of solving some FPK equations in low-dimensions. Thereafter, the EPC method can be employed to solve the FPK equation in low-dimensions. Hence the whole solution procedure is named subspace-EPC method. The solution procedure is presented and numerical examples are given for the systems with various system nonlinearities. The numerical results obtained with the subspace-EPC method are compared with those from MCS and EQL to show the effectiveness of the subspace-EPC method in analyzing various highly nonlinear systems.