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Opinion Dynamics of Sznajd Model on Small-World Network

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Abstract. The society structure plays an important role in shaping the attitudes, beliefs and public opinion. For studying the role of the society structure in opinion dynamics, we analyze the Sznajd model on small-world network formed by adding shortcuts in a lattice consisting of N nodes arranged in a ring and on two-dimensional (2-D) regular lattice. Through computer simulation, we find that there exists a pseudo-phase transition from the coexistence state for $\phi < \phi_c$ to the consensus state for $\phi > \phi_c$, where ϕ_c is some threshold for the shortcut density ϕ, which is dependent of the complex network topology and the dimensionality of complex networks. Our observations indicate the dependence of the opinion dynamics on the complex system topology.

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Key words: Opinion formation, small-world networks, shortcut density, pseudo-phase transition.

1 Introduction

Our local society, which can be well modeled as complex network, has its own structure depending on the geography, culture and history. Recently it has also been realized that many real social networks arising in society, such as networks of collaborations between actors [1, 2] and scientists [3, 4], web-based social networks [5], peer-to-peer social network [6], and the social networks of a bulletin board system in a university [7] all share the small-world effects, including the shortest path length and higher clustering coefficient, which probably caused by the shortcuts in society systems. Those features will affect the dynamics in society systems, especially the opinion dynamics. Many natural

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and man-made networks have been successfully studied as a framework of several celebrated opinion models. Nevertheless, the understanding of the opinion dynamics on complex networks remains a challenge.

The Ising model, one of the fundamental models of Statistical Mechanics, has been modified to model the problem of opinion formation [8–11]. Since many real questions have only two possible answers (yes or no), the Ising model with two possible spins (up or down) is suitable in describing such systems [12]. Following the Ising model and the old principle "united we stand, divided we fall", a celebrated consensus model [8] (Sznajd model) is proposed by K. Sznajd-Weron and J. Sznajd to describe a simple mechanism of making up decisions in a closed community. In the Sznajd model, the opinion S_i of individual i is a binary variable assuming the value +1 (↑) and -1 (↓) that referring to two opposite opinions on a certain thing. Assume that each pair of adjacent individuals can affect the state of their nearest neighbors using the following updating rule:

$$\begin{aligned} &\text{if } S_iS_{i+1}=+1, \text{ then } S_{i-1}=S_{i+2}=S_i, \\ &\text{if } S_iS_{i+1}=-1, \text{ then } S_{i-1}=S_{i+1},\ S_{i+2}=S_i. \end{aligned} \tag{1.1}$$

Simulating the model for the long time, where at each time step the individual i is chosen randomly, one finally obtains one of the three fixed states: ↑↑↑↑, ↓↓↓↓ and ↑↓↑↓, with probabilities, refer to an initially random distribution, 0.25, 0.25 and 0.5, respectively [8,12].

During those years, many physicists have been studied the Sznajd model on two-dimension lattices [9], deterministic pseudo-fractal networks [13], small-world networks [12,14] and scale-free networks [15] through numerical simulation and got some interesting results. However, all those works can not show the important role of the structure of complex systems in opinion formation completely that we will do in our present work. On the other hand, the Sznajd model has been applied in marketing [16, 17] and politics [18–20], and investigated also from the theoretical point of view [21,22].

The main goal of this paper is to analyze the crucial role of complex network topology in opinion dynamics. we find that, by numerical simulation, there exists a long-range opinion-opinion correlation due to the shortcuts added randomly. On the other hand, the probability that there exists a phase transition from the coexistence state to the complete consensus state is one when the shortcut density $\phi > \phi_c$, where ϕ_c is dependent of the complex network size N, its first neighbor parameter (FNP) K and the dimensionality of complex network. Maybe, our present work can explain why a phase transition was not found in [8] and [10], and is helpful for studying the interaction between dynamics and complex system topology.

2 The Sznajd model on complex networks

Many real society systems can be mapped to undirected complex networks, which is a set of agents with relationships of different kinds among them, such as friendship, col-