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## **Fast Inhomogeneous Plane Wave Algorithm for Homogeneous Dielectric Body of Revolution**

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**Abstract.** To solve the electromagnetic scattering problem for homogeneous dielectric bodies of revolution (BOR), a fast inhomogeneous plane wave algorithm is developed. By using the Weyl identity and designing a proper integration path, the aggregation and disaggregation factors can be derived analytically. Compared with the traditional method of moments (MoM), both the memory and CPU time requirements are reduced for large-scale homogeneous dielectric BOR problems. Numerical results are given to demonstrate the validity and the efficiency of the proposed method.

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**Key words**: Bodies of revolution, fast inhomogeneous plane wave algorithm, electromagnetic scattering.

## 1 Introduction

The electromagnetic radiation and scattering from a body of revolution (BOR) of an arbitrary shape have been widely discussed during last several decades. BOR objects of various types, including perfect electric conductors, homogeneous dielectric bodies, coated conducting bodies and combined dielectric and conducting bodies have been studied [1–11]. Because of the axial symmetry of the geometry, only the generatrix that forms the surface of the model is needed for solving the BOR problem in a surface integral equation formulation. Both the memory requirement and the CPU time are reduced compared with the full three-dimensional method [12]. In the BOR integral equation solver, the incident plane wave is expanded in cylindrical modes. The induced current

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of each mode is solved by the integral equation, and finally the radar cross section is obtained by the summation of these modes. The traditional method used to solve a homogeneous dielectric BOR with integral equations is the Method of Moments (MoM) [13], with the memory requirement of  $O(N^2)$ , where *N* is the number of unknowns. It is still time consuming for large-scale BOR problems with the MoM. The computational time consumed in solving the integral equation of the BOR problem mainly depends on the evaluation of modal Green's function (MGF). Much research have been done to solve this problem. Gedney and Mittra used the fast Fourier transform (FFT) to enhance the computational efficiency of the Method of Moments [14]. Abdelmageed and Mohsen used Bartky's transformation and spherical Bessel function expansion to evaluate the modal Green's functions [15,16]. Yu *et al.* also use spherical Bessel functions to expand the MGF and obtain near-axis far-distance closed-form MGFs [17].

In this work, we extend the fast inhomogeneous plane wave algorithm (FIPWA) [18, 19] to accelerate the computation of the MoM for homogeneous dielectric bodies of revolution. PMCHW (Poggio, Miller, Chang, Harrington, Wu) integral equation [20–22] is used for solving the problem of a homogeneous dielectric scatterer excited by plane waves in the free space. The aggregation and disaggregation factors can be computed efficiently due to the cylindric harmonics decomposition. Both the memory requirement and the CPU time are saved for large scale BOR problems. Numerical results are given to demonstrate the validity and efficiency of the FIPWA method. This method also can be used to solve perfect electric conducting (PEC) BOR problems and composite BOR problems with both dielectric and PEC objects.

## 2 Integral equation for body of revolution

## 2.1 PMCHW integral equations for a dielectric BOR object

The scattering of electromagnetic waves from a homogeneous dielectric object having permittivity  $\epsilon_2$  and permeability  $\mu_2$  in a homogeneous background medium ( $\epsilon_1$ , $\mu_1$ ) can be solved by PMCHW (Poggio, Miller, Chang, Harrington, Wu) integral equations as follows:

$$\hat{n} \times \mathbf{E}_{inc} = \hat{n} \times [L_1(\mathbf{J}) + L_2(\mathbf{J})] - \hat{n} \times [K_1(\mathbf{M}) + K_2(\mathbf{M})],$$
(2.1)

$$\hat{n} \times \mathbf{H}_{inc} = \hat{n} \times \left\{ [K_1(\mathbf{J}) + K_2(\mathbf{J})] + \left[ \frac{1}{\eta_1^2} L_1(\mathbf{M}) + \frac{1}{\eta_2^2} L_2(\mathbf{M}) \right] \right\},$$
(2.2)

where **J** and **M** are the induced electric and magnetic current densities,  $\eta_i = \sqrt{\mu_i/\varepsilon_i}$  is the wave impedance for region *i* (*i*=1,2), **E**<sub>*inc*</sub> and **H**<sub>*inc*</sub> are the incident electric field and incident magnetic field, respectively. The operators  $L_i$  and  $K_i$  are defined as

$$L_{i}(\mathbf{x}) = j\omega\mu_{i} \int_{S} \left[ \mathbf{x}G_{i} + \frac{1}{\omega^{2}\mu_{i}\varepsilon_{i}} \nabla\nabla \cdot \mathbf{x}G_{i} \right] ds, \qquad (2.3)$$