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Gyrokinetic Simulation of Magnetic Compressional Modes in General Geometry

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Abstract. A method for gyrokinetic simulation of low frequency (lower than the cyclotron frequency) magnetic compressional modes in general geometry is presented. The gyrokinetic-Maxwell system of equations is expressed fully in terms of the compressional component of the magnetic perturbation, $\delta B_{\|}$, with finite Larmor radius effects. This introduces a "gyro-surface" averaging of $\delta B_{\|}$ in the gyrocenter equations of motion, and similarly in the perpendicular Ampere's law, which takes the form of the perpendicular force balance equation. The resulting system can be numerically implemented by representing the gyro-surface averaging by a discrete sum in the configuration space. For the typical wavelength of interest (on the order of the gyroradius), the gyro-surface averaging can be reduced to averaging along an effective gyro-orbit. The phase space integration in the force balance equation can be approximated by summing over carefully chosen samples in the magnetic moment coordinate, allowing for an efficient numerical implementation.

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1 Introduction

The low frequency compressional magnetic perturbations, $\delta B_{\|}$, are commonly neglected in gyrokinetic simulations [17, 20, 22], which assume a low β plasma (β is the ratio of kinetic pressure to magnetic pressure). As β increases the role of $\delta B_{\|}$ becomes more important due to plasma diamagnetism, and may no longer be neglected. The finite β introduces new modes, such as the magnetic trapped particle mode [26], the drift mirror mode [12], or the drift compressional mode [13], in which $\delta B_{\|}$ is a dominant field

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perturbation. Even in low β plasmas the compressional component of the magnetic perturbation may have to be considered [15].

A numerical difficulty in gyrokinetic simulation of compressional modes is the calculation of finite Larmor radius effects in general geometry. Various numerical techniques have been used in the past to take into account the finite Larmor radius in the gyrokinetic simulations containing nonzero compressional component of the perturbed magnetic field ($\delta B_{\|}$). One of the key features that distinguishes these techniques is how they implement gyro-orbit averaging. In case of low frequency compressional modes (mode frequency smaller than ion cyclotron frequency), the gyro-orbit averaged quantity has been cast into the form of $\left\langle \delta\mathbf{A}_{\perp}\cdot\mathbf{v}_{\perp}\right\rangle$, where $\delta\mathbf{A}_{\perp}$ is the component of the vector potential perpendicular to the equilibrium magnetic field ($\delta B_{\|}=\hat{\mathbf{b}}\cdot\nabla\times\delta\mathbf{A}_{\perp}$), $\mathbf{v}_{\perp}$ is the perpendicular component of the particle velocity, $\hat{\mathbf{b}}$ is the unit vector along the equilibrium magnetic field, and $\langle\cdots\rangle$ stands for the gyro-orbit average. In simulations implementing a pseudo-spectral method in a simple geometry, the gyro-orbit averages may be performed analytically for each spectral component [16]. Another approach has been to perform the gyro-orbit averages explicitly in configurations space, using the method described in [17] and [21] to solve the gyrokinetic Vlasov-Poisson system; and extended in [18] to an electromagnetic case. The method is to average the quantity $\delta\mathbf{A}_{\perp}\cdot\mathbf{v}_{\perp}$ directly in configuration space, over a finite number of samples along the gyro-orbit. Variations of this technique were used in hybrid simulations of energetic particle effects on low frequency MHD modes in [2], and gyrokinetic simulations of the mirror mode in [25].

In the gyrokinetic theory [1,3–7,10,19,24,27], the fundamental operation of gyro-orbit averaging reduces the number of dynamical variables from six, in the particle phase space ($\mathbf{x}$,$\mathbf{v}$), to five in the gyrocenter phase space ($\mathbf{X}$,$v_{\|}$,μ). This, in turn, results in the decrease of the number of independent field quantities appearing in the Maxwell's equations. Thus, the perpendicular Ampere's law has now only one degree of freedom. Adopting the convention of [8], the independent quantities are the electrostatic potential, ϕ; the parallel component of the vector potential, $\delta A_{\|}$; and the compressional component of the magnetic perturbation, $\delta B_{\|}$. It is then undesirable to use $\delta\mathbf{A}_{\perp}$ rather than $\delta B_{\|}$ in gyrokinetic simulation models, from either a numerical or a physical standpoint, since $\delta B_{\|}$ is a scalar and a more physical and fundamental field quantity than $\delta\mathbf{A}_{\perp}$. In the current work, a gyrokinetic system for low frequency compressional modes in general geometry is expressed fully in terms of the compressional component of the magnetic perturbation, $\delta B_{\|}$. This introduces a "gyro-surface" average of $\delta B_{\|}$ in the gyrocenter equations of motion, and similarly in the perpendicular Ampere's law, which takes the form of the low frequency perpendicular force balance equation. The resulting system may be numerically approximated by representing the gyro-surface averaging by a discrete sum in the configuration space. For the typical wavelength of interest (on the order of the gyroradius), the gyro-surface averaging may be reduced to averaging along an effective gyro-orbit, for an efficient numerical implementation. The phase space integration which appears in the low frequency force balance equation, may be approximated by carefully