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Numerical Study of Solutions of the 3D Generalized Kadomtsev-Petviashvili Equations for Long Times

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Abstract. From a spectral method combined with a predictor-corrector scheme, we numerically study the behavior in time of solutions of the three-dimensional generalized Kadomtsev-Petviashvili equations. In a systematic way, the dispersion, the blow-up in finite time, the solitonic behavior and the transverse instabilities are numerically inspected.

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Key words: 3D-KP equations, spectral method, predictor-corrector method, dispersion, blow-up, soliton, transverse instability.

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1 Introduction

The propagation of long, dispersive and weakly nonlinear waves, essentially in the x-direction with weak transverse effects in the y-direction, is modelled by the Kadomtsev-Petviashvili (KP) equation [11],

$$u_t+u_x+uu_x+u_{xxx}+a\partial_x^{-1}u_{yy}=0, \tag{1.1}$$

which is called KP-I if $a=-1$ and KP-II if $a=1$, according to whether the surface tension is neglected or not. The approach in [11] for introducing (1.1) from the Korteweg de Vries (KdV) equation,

$$u_t+u_x+uu_x+u_{xxx}=0,$$

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can be extended to the context of two transverse variables. As explained by Molinet, Saut & Tzvetkov [15], in this context, if we aim at preserving the finite propagation speed properties of the transport operator $\partial_t+\partial_x$ for waves localized in the frequency regions $|\xi_2|/|\xi_1|\ll 1$ and $|\xi_3|/|\xi_1|\ll 1$, where ξ_1, ξ_2 and ξ_3 are the Fourier modes corresponding respectively to the space variables x, y and z, we are led to consider the generalized operator

$$\partial_t+\partial_x+\frac{1}{2}\left(\partial_x^{-1}\partial_{yy}+\partial_x^{-1}\partial_{zz}\right),$$

where ∂_x^{-1} denotes the anti-derivative, also defined such that

$$\widehat{\partial_x^{-1}u}(\xi_1,\xi_2,\xi_3):=\frac{\widehat{u}(\xi_1,\xi_2,\xi_3)}{\mathrm{i}\xi_1},$$

and $\widehat{u}$ represents the Fourier transform of u.

In this paper, we are concerned with the generalized Kadomtsev-Petviashvili equations in three-dimensional space:

$$u_t+u^p u_x+u_{xxx}+a\partial_x^{-1}u_{yy}+b\partial_x^{-1}u_{zz}=0, \tag{1.2}$$

where $p\geq 1$, and the constants a, b are normalized to ± 1. The separate term u_x does not of course appear in (1.2) since a change of functions has now been applied ($u(x,y,z,t):=\widetilde{u}(x+t,y,z,t)$). The mass and the energy,

$$\int_{\mathbb{R}^3} u^2(x,y,z,t)\,dxdydz$$

and

$$\int_{\mathbb{R}^3}\left[\frac{u^{p+2}}{(p+1)(p+2)}-\frac{1}{2}u_x^2+\frac{a}{2}(\partial_x^{-1}u_y)^2+\frac{b}{2}(\partial_x^{-1}u_z)^2\right](x,y,z,t)\,dxdydz,$$

are conserved by the flow associated with (1.2). Although the literature proposes an extensive list of (theoretical and numerical) works related with the KP equations in dimension two, namely with the Cauchy problem based on (1.1), see, e.g., [3,7–10,12,18,22,25], we do not find the same range of references in the three-dimensional case. Some theoretical results concerning the behavior of solutions of (1.2) have been recently given in [4,5,13,19]. In particular, the existence as well as the non-existence of solitary waves are proved by De Bouard & Saut [4,5], and Saut establishes in [19] a result of blow-up in finite time for $a=b=-1$ and $p\geq 2$. In [13], Liu continues this investigation and proves that, when a solution of (1.2), for $a=b=-1$ and $1\leq p<4/3$, is initially close to an unstable solitary wave, then this solution blows up in finite time. Since no result has yet been proved for blow-up in finite time, with $a=b=1$ or $ab=-1$, there is considerable interest in performing numerical simulations aimed at studying the solutions of (1.2) in various situations.

The aim of this paper consists not only of inspecting numerically certain theoretical properties already stated, but above all of investigating, in diverse contexts, aspects not