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An Implicit LU-SGS Scheme for the Spectral Volume Method on Unstructured Tetrahedral Grids

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Abstract. An efficient implicit lower-upper symmetric Gauss-Seidel (LU-SGS) solution approach has been applied to a high order spectral volume (SV) method for unstructured tetrahedral grids. The LU-SGS solver is preconditioned by the block element matrix, and the system of equations is then solved with a LU decomposition. The compact feature of SV reconstruction facilitates the efficient solution algorithm even for high order discretizations. The developed implicit solver has shown more than an order of magnitude of speed-up relative to the Runge-Kutta explicit scheme for typical inviscid and viscous problems. A convergence to a high order solution for high Reynolds number transonic flow over a 3D wing with a one equation turbulence model is also indicated.

AMS subject classifications: 65M70, 76M12, 76M22

Key words: High-order, unstructured grid, spectral volume, implicit method.

1 Introduction

Unstructured grid methods have been widely used in aerodynamic design processes because they can offer flexible grid generation for 3D complex configurations. Aerodynamic coefficients at cruising condition can be predicted reasonably well with steady Reynolds Averaged Navier-Stokes Simulation (RANS) using conventional finite volume solvers of second order accuracy in space. However, some problems become prohibitively expensive to reach sufficient prediction accuracy with increasing grid points. Examples include vortex dominated flows such as flow over high-lift configurations, aero-acoustic noise predictions and LES/DNS for high Reynolds number flowfields. In these flows, in order to resolve important flow features of unsteady vortices, high-order methods are required.

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In a conventional finite volume framework, arbitrarily high-order schemes can be obtained theoretically by extending stencils for high-order polynomial data reconstructions, but a crucial difficulty exists in finding valid (non-singular) stencils for unstructured grids. Recently, new high-order unstructured grid methods such as the discontinuous Galerkin (DG) method [5,6], the spectral volume (SV) method [9,15,23,27–29,31] and the spectral difference (SD) method [14] have attracted attention. These methods introduce degrees of freedom within each element for a high-order approximation to the solution and are expected to achieve the formal order of accuracy even on an unstructured grid.

The SV method belongs in a class of Godunov-type finite volume methods, and yet the degrees-of-freedom are introduced by partitioning a grid cell. If a proper partition is found, a valid reconstruction stencil comprised of partitioned sub-cells can be predetermined. In our previous study [8], the SV method was developed for the Euler and the Navier-Stokes equations in 3D and we demonstrated its applicability to large-scale parallel computations. However, the developed solution technique was CPU demanding and inefficient for steady state problems. The excessively high CPU cost was attributed mainly to the use of an explicit Runge-Kutta time integration scheme. The Runge-Kutta scheme is easy to implement and requires only a small amount of memory, but suffers from slow convergence, especially for viscous problems in which grid points are clustered in the boundary layer.

Several efficient time integration/iterative solution approaches for the DG method have been developed, for example, preconditioned GMRES approaches [2,17], a p multigrid approach using a line implicit smoother [7] and a low storage p multigrid approach [16]. Recently, an efficient LU-SGS scheme [4] has been developed for the SD method [24]. Implicit schemes for the SV method on tetrahedral grids, however, are yet to be developed.

In this study, the LU-SGS approach is applied to the SV method on unstructured tetrahedral grids. The LU-SGS solver is preconditioned by the block SV-element matrix, and the system of equations is then solved with an exact LU decomposition linear solver. The developed method is tested first for a linear advection problem, and then for typical steady problems by solving the Euler and Navier-Stokes equations. A significant reduction in computational time comparing with the multi-stage Runge-Kutta scheme is demonstrated. Finally, we show the converged solution of viscous transonic flow over a 3D wing, where the effect of turbulence is accounted for by solving a one-equation turbulence model using a SV scheme, to indicate the applicability of the present implicit SV scheme for practical engineering problems.

2 Numerical methods

2.1 Spectral volume discretization

The unsteady, 3D, compressible Navier-Stokes equations in conservative form can be expressed as