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Numerical Study of Vortex Interactions in Bose-Einstein Condensation

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Abstract. The dynamics and interaction of quantized vortices in Bose-Einstein condensates (BECs) are investigated by using the two-dimensional Gross-Pitaevskii equation (GPE) with/without an angular momentum rotation term. If all vortices have the same winding number, they would rotate around the trap center but never collide. In contrast, if the winding numbers are different, their interaction highly depends on the initial distance between vortex centers. The analytical results are presented to describe the dynamics of the vortex centers when $\beta=0$. While if $\beta\neq 0$, there is no analytical result but some conclusive numerical findings are provided for the further understanding of vortex interaction in BECs. Finally, the dynamic laws describing the relation of vortex interaction in nonrotating and rotating BECs are presented.

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Key words: Rotating Bose-Einstein condensation, Gross-Pitaevskii equation, angular momentum rotation, vortex interaction, vortex pair, vortex dipole.

1 Introduction

The first observation of a single vortex line in weakly interacting alkali gases has verified the superfluid properties of Bose-Einstein condensates (BECs) [1]. Recently, many efforts have been made to develop more complicated vortices. For example, vortex lattices containing a large number of vortices were created by rotating the system with a laser spoon [2, 3]; multiply charged vortices were also observed by using the topological phase engineering methods [4]. It is expected that more complicated vortex clusters can be created in the future with the further development of phase imprinting method. The

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achievement of vortex states would enable various opportunities, ranging from investigating the properties of random polynomials [5] to using vortices in quantum memories [6]. All of these developments have stirred interests in the study of states with several vortices.

So far, there have been a number of investigations on the properties of vortices in BECs. For example, the manipulation of the topological charge of a vortex by external potentials was discussed in [7]; different stationary vortex cluster states were observed in nonrotating BECs [8–10], and later the dynamical stability of these cluster was investigated in [11]. The interaction of vortices in nonrotating BECs was studied in [12] when the atomic interaction is very weak. The generation and dynamics of vortex-antivortex pairs were studied in a toroidal condensate [13]. By using the Thomas-Fermi approximation, analytical expressions for the angular momentum and the energy of a vortex dipole were obtained in [14].

The main aims of this paper are: i) to provide a detailed study of the vortex interaction in nonrotating BECs; ii) to extend the interaction study from nonrotating BECs to rotating condensates. There are four possible reasons affecting the vortex interaction: the winding number of a vortex, the initial distance between vortex centers, the strength of atomic interaction, and the angular rotation speed if in rotating BECs. To get an insight about the vortex interaction, we start by considering the interaction with zero atomic interaction, and find the analytical results to describe the motion of vortex centers. This study is extended by taking a small atomic interaction into account, and due to the nonlinear effect the dynamics become more complicated. To the best of our knowledge, there is still no study about the interaction of vortices in strongly interacting BECs. In this paper, we obtain some conclusive findings for the strongly interacting case, which may provide the further understanding of the vortex interaction in BECs. Finally, the vortex interaction in rotating BECs is studied. Analytical results are derived to show the relation between the interaction in nonrotating and rotating BECs. Some numerical results are also presented to verify our analytical findings.

The paper is organized as follows. In Section 2, the mathematical model and numerical schemes are introduced. In Section 3, the dynamics and interaction of one, two and more vortices are investigated in detail for nonrotating BECs. It is generalized to the rotating BECs in Section 4, and also a relation connecting the dynamics of vortices in nonrotating and rotating BECs is presented in Section 4. Section 5 provides a summary and brief discussion.

2 The mathematical model and numerical schemes

2.1 The mathematical model

At a temperature T much smaller than the critical temperature T_c, the properties of BECs in a rotational frame can be well described by the macroscopic wave function $\psi(\mathbf{x},t)$, whose evolution is governed by a self-consistent, mean field nonlinear Schrödinger equa-