

## Application of Lattice Boltzmann Method to Simulation of Compressible Turbulent Flow

Congshan Zhuo<sup>1</sup>, Chengwen Zhong<sup>1,2</sup>, Kai Li<sup>1</sup>, Shengwei Xiong<sup>1</sup>, Xiaopeng Chen<sup>3</sup> and Jun Cao<sup>4,\*</sup>

<sup>1</sup> National Key Laboratory of Science and Technology on Aerodynamic Design and Research, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China.

<sup>2</sup> Center for High Performance Computing, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China.

<sup>3</sup> School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an, Shaanxi 710072, China.

<sup>4</sup> Department of Mechanical and Industrial Engineering, Ryerson University, Toronto, Ontario, M5B 2K3, Canada.

Received 30 January 2010; Accepted (in revised version) 7 May 2010

Communicated by Rho Shin Myong

Available online 23 June 2010

---

**Abstract.** The main goal of this paper is to develop the coupled double-distribution-function (DDF) lattice Boltzmann method (LBM) for simulation of subsonic and transonic turbulent flows. In the present study, we adopt the second-order implicit-explicit (IMEX) Runge-Kutta schemes for time discretization and the Non-Oscillatory and Non-Free-Parameters Dissipative (NND) finite difference scheme for space discretization. The Sutherland's law is used for expressing the viscosity of the fluid due to considerable temperature change. Also, the Spalart-Allmaras (SA) turbulence model is incorporated in order for the turbulent flow effect to be pronounced. Numerical experiments are performed on different turbulent compressible flows around a NACA0012 airfoil with body-fitted grid. Our numerical results are found to be in good agreement with experiment data and/or other numerical solutions, demonstrating the applicability of the method presented in this study to simulations of both subsonic and transonic turbulent flows.

**AMS subject classifications:** 76M28, 82B40, 76N15, 76F55

**Key words:** Lattice Boltzmann method, compressible turbulent flow, airfoil, body-fitted grid.

---

\*Corresponding author. *Email addresses:* zhuocs@mail.nwpu.edu.cn (C. Zhuo), zhongcw@nwpu.edu.cn (C. Zhong), li040384@mail.nwpu.edu.cn (K. Li), xiongshengwei@mail.nwpu.edu.cn (S. Xiong), xchen76@nwpu.edu.cn (X. Chen), jcao@ryerson.ca (J. Cao)

## 1 Introduction

The Lattice Boltzmann method (LBM) has recently attracted an increasing amount of attention from the computational fluid dynamics (CFD) community [1, 2]. The LBM is a derivative of the lattice gas automata (LGA) method with some advantages of the LGA successfully inherited. Different from the conventional numerical methods for solving the macroscopic governing equations, the LBM is based on microscopic models and the kinetic theories [3]. The mechanism of LBM is parallel in nature due to the locality of particle interaction and the transport of particle information, so it is well suitable for massively parallel computing. Moreover, the LBM has some other advantages, such as good numerical robustness, flexibility with respect to complex boundaries, and computational efficiency.

As of today, the LBM has achieved great success in simulating semi-incompressible and isothermal fluid flows. Available literatures also reveal that the LBM has been successfully applied to the solution of the Euler [4–7] or Navier-Stokes (N-S) equations [8–22] for a compressible fluid. For instance, Qu et al. [7] proposed a non-free-parameter LBM to construct equilibrium distribution functions for inviscid compressible flows at high Mach number. Sun et al. [10–13] developed a locally adaptive lattice Boltzmann model suitable for flows in a wide range of Mach numbers for compressible flows. Watari [15] proposed finite difference lattice Boltzmann method (FDLBM) for numerical simulations of flows from subsonic to supersonic ranges for both inviscid (Euler model) and viscous (Navier-Stokes model) fluids. Yan et al. [4, 17] presented a compressible LBM with three-speed and three-energy-level for the Euler [4] and Navier-Stokes equations [17]. Pan et al. [18] and Gan et al. [19] also worked on improving lattice Boltzmann model for some high-speed inviscid and viscous supersonic flow cases, respectively, with higher Mach numbers (up to 30 or a bit above). Recently, Li Q [20], Wang Y et al. [21] developed a coupled double-distribution-function (DDF) LBM by combining the DDF approach and the multi-speed approach, and used it to simulate compressible fluid flow with arbitrary specific-heat ratio and Prandtl number. In this method, a density distribution function based on a multi-speed lattice as well as a total energy distribution function are used, and these two distribution functions are coupled together via the state equation. In [20], Li Q et al. applied two different methods to construct equilibrium distribution functions in two coupled DDF models, respectively. Model 1 is based on the truncated Maxwellian distribution function and limited to low- and moderate-Mach-number viscous fluid flows; Model 2 is based on a circular function [7] and can be used to simulate viscous fluid flows with high Mach numbers. In the two models, the density distribution function is used to recover the compressible continuity and momentum equations, while the energy equation is recovered by a total energy distribution function. The total energy distribution function is coupled with the density distribution function via the ideal gas law. This method can be used for non-uniform grid through the transformation of coordinates [22].

Most flows encountered in engineering applications are of turbulent nature. The pre-