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A Weighted Multi-Domain Spectral Penalty Method with Inhomogeneous Grid for Supersonic Injective Cavity Flows

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Abstract. In [*J. Comput. Phys.* **192(1)**, pp.325-354 (2003)], we have developed a multi-domain spectral method with stable and conservative penalty interface conditions for the numerical simulation of supersonic reactive recessed cavity flows with homogeneous grid. In this work, the previously developed methodology is generalized to inhomogeneous grid to simulate the two dimensional supersonic injector-cavity system. Non-physical modes in the solution generated at the domain interfaces due to the spatial grid inhomogeneity are minimized using the new weighted multi-domain spectral penalty method. The proposed method yields accurate and stable solutions of the injector-cavity system which agree well with experiments qualitatively. Through the direct numerical simulation of the injector-cavity system using the weighted method, the geometric effect of the cavity wall on pressure fluctuations is investigated. It is shown that the recessed slanted cavity attenuates pressure fluctuations inside cavity enabling the cavity to act potentially as a stable flameholder for scramjet engine.

AMS subject classifications: 65M12, 65M70, 76J20

Key words: Penalty interface conditions, weighted multi-domain spectral penalty methods, supersonic recessed cavity flame-holder, compressible Navier-Stokes equations.

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1 Introduction

Spectral methods have been actively used in the computational fluid dynamics community in the last decades due to the merit of high order accuracy maintained for long time

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integration. Spectral methods have been also applied to highly complex fluid systems and have been proven to yield accurate solutions even with the stiff or discontinuous spatial gradients. These systems include the supersonic shock bubble interactions [12], the supersonic cavity flows [11], the Ritchmyer-Meshkov instability [20, 21] and etc. The difficulty of implementing the spectral method to these complex fluid systems is to deal with the nonsmooth spatial gradients successfully. The discontinuous solution is commonly found in most high speed fluid mechanical systems. The spectral approximation of such solution yields spurious oscillations near the discontinuity, known as the Gibbs phenomenon. These Gibbs oscillations deteriorate both the accuracy and stability in general. The essential methodology to deal with such oscillations in the spectral solution is the spectral viscosity or filtering methods [5, 16, 22, 25]. The filtering which is mathematically equivalent to the spectral viscosity method but practically more efficient, is used to stabilize the flow fields over the time integration. The filtering reduces the high order oscillations by attenuating the high modes in the solution. The filtering method can be applied either globally or locally. By applying the filtering locally one can obtain more accurate solution in the smooth region. Thus it is desirable to separate the locally nonsmooth regions from the global smooth region. A multi-domain spectral method has been developed to address this problem [8, 11, 13, 14, 17–19], with which the physical domain is split into multiple subdomains. For the multi-domain spectral method, the proper interface conditions should be imposed at the domain interfaces. The simplest condition is the averaging method. With the averaging method, the flow field at the domain interface is obtained by averaging the two adjacent solutions across the interface. Thus the continuity of the solution is ensured with the averaging method. Although this method is simple and efficient to be implemented, it may cause the generation of nonphysical solutions at the interface if the two adjacent subdomains have different grid resolutions near the interface, i.e. if the grid system is inhomogeneous. We define the grid *inhomogeneity* as the grid configuration such that the grid resolutions between the adjacent subdomains across the domain interface are different. Such difference can be obtained by having each domain have either different order of polynomials or different domain length. If the grid distribution is inhomogeneous, the stable interface conditions derived for the homogeneous grid system are not enough and one needs to find the conditions with which the spatial inhomogeneity can be addressed properly.

At the domain interface of two adjacent subdomains which have the degree of polynomials, N_1 and N_2, and the domain lengths Δ^I and Δ^{II} in the x-direction, respectively, the ratio of the grid spacing between Δx_1 and Δx_2 at the interface is approximately given by

$$\frac{\Delta x_2}{\Delta x_1} = \frac{\Delta^{II}}{\Delta^I} \cdot \frac{N_1^2}{N_2^2}. \tag{1.1}$$

If the grid spacing ratio $\Delta x_2 / \Delta x_1$ is different and far from unity, we consider it as the inhomogeneous grid system. If $\Delta x_2 / \Delta x_1 = 1$, the grid is homogeneous, and the averaging method can play an efficient role as a stable interface condition. However, if the ratio