

A Hybrid Algorithm of Event-Driven and Time-Driven Methods for Simulations of Granular Flows

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Abstract. The classical discrete element approach (DEM) based on Newtonian dynamics can be divided into two major groups, event-driven methods (EDM) and time-driven methods (TDM). Generally speaking, TDM simulations are suited for cases with high volume fractions where there are collisions between multiple objects. EDM simulations are suited for cases with low volume fractions from the viewpoint of CPU time. A method combining EDM and TDM called Hybrid Algorithm of event-driven and time-driven methods (HAET) is presented in this paper. The HAET method employs TDM for the areas with high volume fractions and EDM for the remaining areas with low volume fractions. It can decrease the CPU time for simulating granular flows with strongly non-uniform volume fractions. In addition, a modified EDM algorithm using a constant time as the lower time step limit is presented. Finally, an example is presented to demonstrate the hybrid algorithm.

AMS subject classifications: 70F05, 65K05, 74S30, 70F10

Key words: Time-driven method (TDM), event-driven method (EDM), Hybrid algorithm of event-driven and time-driven methods (HAET), granular flow.

1 Introduction

Granular materials are very commonly used in industry and in our daily life. Yet many aspects of granular flows are poorly understood. Granular materials are complex systems composed of a very large number of solid particles. The motion of each particle is defined by classical Newtonian mechanics and contact mechanics. Based on the relative velocity between particles, granular material is classified into three phases, as gas-like, liquid-like and solid-like [1]. Important parameters used to determine the phase are

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the volume fraction ϕ and coefficients of restitution (COR) [2]. With respect to the flow cases and the accuracy required, various methods can be used to simulate the motion of granular materials numerically. According to Hogue and Newland [3], the methods are classified into two approaches: continuum mechanics methods (CMM) and discrete element methods (DEM). The CMM uses the Eulerian approach to simulate granular material behavior. Several reviews of CMM are available [4–6]. The DEM is based on the Lagrangian approach to simulate the motion of each particle at the microscopic scale. It can be divided into three main classes [4]: *Statistical mechanics models* [7], *Newtonian dynamics models*, and *Hybrid models* [3]. Furthermore, the Newtonian dynamics models can be divided into two major groups, *Event-driven methods* (EDM) and *Time-driven methods* (TDM). A detailed review of the Newtonian dynamics models is presented in the next sections. This paper presents a new hybrid algorithm EDM and TDM. It can be used for strongly non-uniform flow field with a higher efficiency than TDM. In this hybrid algorithm, the choice of using either EDM or TDM depends on the local volume fraction. Hence it is required that the velocity changes determined from these two models should be close to each other for the same binary collision. To characterize the velocity changes, two COR, e and β are introduced for the normal direction and the tangential direction respectively. The definitions of these two coefficients are expressed as the ratio of the post-collisional relative velocities over the pre-collisional relative velocities at the contact point, given by

$$e = -\frac{\mathbf{k} \cdot \mathbf{u}_{ij}^{lo}}{\mathbf{k} \cdot \mathbf{u}_{ij}^o}, \quad (1.1a)$$

$$\beta = -\frac{\mathbf{k} \times \mathbf{u}_{ij}^{lo}}{\mathbf{k} \times \mathbf{u}_{ij}^o}, \quad (1.1b)$$

where \mathbf{k} is the unit vector along the center line from particle i to j and \mathbf{u}_{ij}^o is the relative velocity at the contact point. In this paper, vector variables are represented by bold font and scalar variables are represented by italic font. The ranges of the two COR are $0 \leq e \leq 1$, and $-1 \leq \beta \leq 1$ respectively.

2 Time-driven method

The calculation of the contact force between each discrete element in the time-driven method used in the HAET algorithm is based on the original method of Cundall and Strack [8]. An integration method, which is introduced in Section 4, can be employed to calculate the changes in velocity and position for each particle after a certain time by considering all the forces on the particle. Some of the forces acting between particles originate from the deformation of the particles when they are in contact with their neighbors, as shown in the Fig. 1. Ramirez [9] suggested that the normal particle-particle collision process could be modeled as a spring-dashpot system, where the normal force, F_n , is the