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A TVD Uncertainty Quantification Method with Bounded Error Applied to Transonic Airfoil Flutter

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Abstract. The Unsteady Adaptive Stochastic Finite Elements (UASFE) approach is a robust and efficient uncertainty quantification method for resolving the effect of random parameters in unsteady simulations. In this paper, it is shown that the underlying Adaptive Stochastic Finite Elements (ASFE) method for steady problems based on Newton-Cotes quadrature in simplex elements is extrema diminishing (ED). It is also shown that the method is total variation diminishing (TVD) for one random parameter and for multiple random parameters for first degree Newton-Cotes quadrature. It is proven that the interpolation of oscillatory samples at constant phase in the UASFE method for unsteady problems results in a bounded error as function of the phase for periodic responses and under certain conditions also in a bounded error in time. The two methods are applied to a steady transonic airfoil flow and a transonic airfoil flutter problem.

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Key words: Total variation diminishing, extrema diminishing, error bounds, stochastic finite elements, uncertainty quantification, transonic flow, transonic flutter.

1 Introduction

Deterministic numerical solutions of engineering flow and fluid-structure interaction problems contain no information about the influence of parameter variations on the outputs of interest. Physical uncertainties are, however, present in practically all engineering applications due to, for example, varying atmospheric conditions, and production

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tolerances affecting material properties and the geometry. These inherent physical variations enter the computational problem through physical input parameters, and initial and boundary conditions. Especially, discontinuous solutions of shock waves in supersonic flow and bifurcation phenomena of aeroelastic systems are highly sensitive to this input variability. Dynamic fluid-structure interaction systems also amplify input variations with time.

Physical variability is here described in a probabilistic framework by random parameters with known probability density. The distribution functions and the statistical moments of outputs of interest are determined in order to obtain more reliable computational predictions, which can be utilized in robust design optimization and reducing design safety factors. In contrast, in structural reliability analysis input randomness is propagated to compute the probability of failure [4]. Failure probabilities are often small such that in that case the tails of the distribution are of interest.

The resulting mathematical formulation of the uncertainty quantification problem for output of interest $u(\mathbf{x},t,\omega)$ is

$$\mathcal{L}(\mathbf{x},t,\omega;u(\mathbf{x},t,\omega))=\mathcal{S}(\mathbf{x},t,\omega), \tag{1.1}$$

with appropriate initial and boundary conditions. Operator $\mathcal{L}$ and source term $\mathcal{S}$ are defined on domain $D\times T\times\Omega$, where $\mathbf{x}\in D$ and $t\in T$ are the spatial and temporal dimensions with $D\subset\mathbb{R}^d$, $d=\{1,2,3\}$, and $T\subset\mathbb{R}$. The argument ω emphasizes that $u(\mathbf{x},t,\omega)$ is a random event with the set of outcomes Ω of the probability space (Ω, $\mathcal{F}$, P) with $\mathcal{F}\subset 2^\Omega$ the σ-algebra of events and P a probability measure. The probability space originates from n_a uncorrelated second order random parameters

$$\mathbf{a}(\omega)=\{a_1(\omega),\cdots,a_{n_\mathrm{a}}(\omega)\}\in A,$$

with probability density $f_\mathbf{a}(\mathbf{a})$ in Eq. (1.1) and its initial and boundary conditions, with parameter space $A\subset\mathbb{R}^{n_\mathrm{a}}$.

For a single realization $\omega=\omega_k$, $u(\mathbf{x},t,\omega_k)$ reduces to the deterministic function $u_k(\mathbf{x},t)$ in terms of the spatial coordinates $\mathbf{x}$ and time t. The numerical approximation of $u_k(\mathbf{x},t)$ can be obtained using standard spatial discretization methods and time marching schemes. A weighted approximation of the response surface $u^*(\mathbf{x},t,\mathbf{a})$ based on n_s deterministic solutions $\{u_k(\mathbf{x},t)\}_{k=1}^{n_\mathrm{s}}$ is considered a solution of uncertainty quantification problem (1.1). Integration and sorting of $u^*(\mathbf{x},t,\mathbf{a})$ results in the statistical moments $\mu_{\mathrm{u}_i}(\mathbf{x},t)$

$$\mu_{\mathrm{u}_i}(\mathbf{x},t)=\int_A u^*(\mathbf{x},t,\mathbf{a})^i f_\mathbf{a}(\mathbf{a})\mathrm{d}\mathbf{a}, \tag{1.2}$$

and its probability distribution.

The classical approach of solving (1.1) by computing many deterministic solutions for randomly sampled parameter values in a Monte Carlo simulation [9] leads to impractically high computational costs for flow and fluid-structure simulations, which are already computationally intensive in the deterministic case. Non-intrusive Polynomial