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Central Schemes and Second Order Boundary Conditions for 1D Interface and Piston Problems in Lagrangian Coordinates

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Abstract. We study high-resolution central schemes in Lagrangian coordinates for the one-dimensional system of conservation laws describing the evolution of two gases in slab geometry separated by an interface. By using Lagrangian coordinates, the interface is transformed to a fixed coordinate in the computational domain and, as a consequence, the movement of the interface is obtained as a byproduct of the numerical solution. The main contribution is the derivation of a special equation of state to be imposed at the interface in order to avoid non-physical oscillations. Suitable boundary conditions at the piston that guarantee second order convergence are described.
We compare the solution of the piston problem to other results available in the literature and to a reference solution obtained within the adiabatic approximation. A shock-interface interaction problem is also treated. The results on these tests are in good agreement with those obtained by other methods.

AMS subject classifications: 65M06, 65M99, 76T05

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1 Introduction

The numerical simulation of interface and multi-fluid problems is a research topic of relevant interest in several branches of the applied sciences. To mention some issues, we

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have: bubble evolution in nuclear flows, combustion applications, chemical reactions, etc.

We study high-resolution central schemes for the one-dimensional system of conservation laws describing the evolution of two gases in slab geometry separated by an interface, using the equations in Lagrangian form. By using Lagrangian coordinates, the interface is transformed to a fixed coordinate in the computational domain and, as a consequence, the movement of the interface is obtained as a byproduct of the numerical solution. The idea to apply a Lagrangian method to interface problems is not new and for a survey on this subject we refer to the one by Benson [4].

The difficulties to retain pressure equilibrium at the interface has been indicated as the main reason for the failure of many successful single component schemes when applied to multi-component problems by several authors (see the review article by Abgrall and Karni [3] and the references quoted therein). A solution to this problem has been to use the pressure equation in the cells near the interface, as proposed by Karni [13] and investigated by Abgrall [2]. In this way the scheme is not entirely conservative, but lack of conservation, that can be checked *a posteriori*, is quite small. A similar approach has been used by Fedkiw *et al.* [10] in the context of multi-fluid flows, see also the book by Osher and Fedkiw [21, Ch. 15] and the references quoted therein, and by Wang *et al.* [23] within a mass conservation principle. Recent studies, for instance by Brummelen and Koren [22] or by Lombard and Donat [18], have proposed conservative and pressure-invariant methods enforcing zero or even first order jump conditions at the interface. The intrinsic difficulty related to the evolution of the interface has induced some authors, Davis [5] and Fazio and LeVeque [8], to apply an exact Riemann solver at the interface to find an approximation of its velocity. In particular, in paper [8] the authors use a moving mesh to enforce that the interface and the piston lie at cell edges.

Here we propose the use of a central scheme on a staggered grid, which is conservative and does not require the solution to exact or approximate Riemann problems. Furthermore, by using Lagrangian coordinates, we do not require the use of additional equations for the interface motion, as is done, for example, with level set methods (see Mulder et al. [19]) or mass fraction models (see Abgrall [1] and Larouturou [16]). The choice of the staggered grid is motivated by the higher resolution of such schemes with respect to the corresponding non-staggered version of the same order [15]. Of course it is possible to improve the resolution of second order central schemes on non staggered grids, but this requires more sophisticated approaches [14].

The choice of a staggered central scheme for the treatment of an interface may be questionable, because of the unavoidable smearing introduced by the staggered approach, as opposed to non staggered upwind schemes that provide a much better resolution of the interface. However, here we want to show that with a suitable choice of an interface condition, staggered central schemes are robust enough to be able to treat interfaces, maintaining a reasonable resolution.

Our scheme is based on the derivation of a suitable equation of state for the pressure to be used in the cell containing the interface. Preliminary work on this subject has been