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Efficient Implicit Non-linear LU-SGS Approach for Compressible Flow Computation Using High-Order Spectral Difference Method

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Abstract. An implicit non-linear lower-upper symmetric Gauss-Seidel (LU-SGS) solution algorithm has been developed for a high-order spectral difference Navier-Stokes solver on unstructured hexahedral grids. The non-linear LU-SGS solver is preconditioned by a block element matrix, and the system of equations is then solved with the LU decomposition approach. The large sparse Jacobian matrix is computed numerically, resulting in extremely simple operations for arbitrarily complex residual operators. Several inviscid and viscous test cases were performed to evaluate the performance. The implicit solver has shown speedup of 1 to 2 orders of magnitude over the multi-stage Runge-Kutta time integration scheme.

AMS subject classifications: 65M70, 76M20, 76M22

Key words: High order, unstructured grids, spectral difference, Navier-Stokes, implicit.

1 Introduction

The past three decades have seen tremendous development in computational fluid dynamics (CFD) as a discipline. CFD is now used routinely to complement the wind tunnel in engineering design. Nearly all production flow solvers are based on second-order numerical methods. They are capable of delivering design-quality Reynolds Averaged Navier-Stokes results with several million cells (degrees of freedom or DOFs) on commercial Beowulf clusters within a few hours.

As impressive as these second order codes are, there are still many flow problems considered as out of reach, e.g., vortex dominated flows including helicopter blade vortex interaction and flows over high-lift configurations. Unsteady propagating vortices are the

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main features of these flow problems, and second-order methods are usually too dissipative to resolve those unsteady vortices for a significant distance. The advantage of high-order methods (order of accuracy >2) over first- and second-order ones is well-known in the CFD community. Generally speaking, with the same number of DOFs or solution unknowns, high-order methods are capable of producing much more accurate results. For problems requiring very high accuracy, e.g., wave propagation problems in computational aeroacoustics, high-order methods have been the main choice. Many high-order methods were developed for structured grids, e.g., ENO/WENO methods [28], compact methods [16, 38], optimized methods [34], to name just a few. In the last two decades, there have been intensive research efforts on high-order methods for unstructured grids since many real world applications have complex geometries. An incomplete list of notable examples includes the spectral element method [22], multi-domain spectral method [14,15], k-exact finite volume method [2,10,17,24], WENO methods [11], discontinuous Galerkin (DG) method [3,7,8], high-order residual distribution methods [1], spectral volume (SV) [20,31,39,41,42] and spectral difference (SD) methods [12,18,19,32,33,43]. Among those methods, some are based on the weighted residual form of the governing equations, for instance, the DG method. Some are based on the integral form of the governing equations, e.g., the k-exact finite volume method and SV methods. Others, such as the staggered grid multi-domain spectral method and the SD method, are based on the differential form. In fact, the staggered-grid multi-domain spectral method and the SD method are identical on quadrilateral or hexahedral grids. More comprehensive reviews of high-order methods are given in [9, 40].

When one chooses a particular method for three-dimensional applications, the cost and the complexity in implementing the method is often an important factor. It is obvious that methods based on the differential form are the easiest to implement since they do not involve surface or volume integrals. This is particularly true when high-order curved boundaries need to be dealt with. We recently developed a high order SD method [32, 33] for the three dimensional Navier-Stokes equations on unstructured hexahedral grids. High-order of accuracy and spectral convergence are achieved for several benchmark problems. It was also shown that the wall boundaries must be approximated with high-order surfaces. An explicit Runge-Kutta time integration scheme [27] was used in the implementation. Although the explicit scheme is easy to implement and has high-order accuracy in time, it suffered from slow convergence, especially for viscous grids which are clustered in the viscous boundary layer. It is well-known that high-order methods are restricted to a smaller CFL number than low order ones. In addition, they also possess much less numerical dissipation. Therefore it takes excessive CPU to reach a state-steady solution with explicit high-order schemes. The computation cost of high-order explicit methods for many steady-state problems is so high that they become less efficient than low-order implicit methods in terms of the total CPU time given the same level of solution error. It is therefore imperative to develop efficient implicit solution approaches for high-order methods to fully realize the potentials, which is the objective of the present study.

Implicit time-integration schemes are highly desired for improved efficiency since