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Critical Behaviour of the Ising S=1/2 and S=1 Model on (3,4,6,4) and (3,3,3,3,6) Archimedean Lattices

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Abstract. We investigate the critical properties of the Ising $S=1/2$ and $S=1$ model on (3,4,6,4) and (3^4,6) Archimedean lattices. The system is studied through the extensive Monte Carlo simulations. We calculate the critical temperature as well as the critical point exponents γ/ν, β/ν, and ν basing on finite size scaling analysis. The calculated values of the critical temperature for S=1 are $k_B T_C/J$=1.590(3), and $k_B T_C/J$=2.100(4) for (3,4,6,4) and (3^4,6) Archimedean lattices, respectively. The critical exponents β/ν, γ/ν, and $1/\nu$, for S=1 are β/ν=0.180(20), γ/ν=1.46(8), and $1/\nu$=0.83(5), for (3,4,6,4) and 0.103(8), 1.44(8), and 0.94(5), for (3^4,6) Archimedean lattices. Obtained results differ from the Ising $S=1/2$ model on (3,4,6,4), (3^4,6) and square lattice. The evaluated effective dimensionality of the system for $S=1$ are $D_{\text{eff}}=1.82(4)$, for (3,4,6,4), and $D_{\text{eff}}=1.64(5)$ for (3^4,6).

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Key words: Monte Carlo simulation, Ising model, critical exponents.

1 Introduction

The Ising model [1, 2] has been used during long time as a "toy model" for diverse objectives, as to test and to improve new algorithms and methods of high precision for calculation of critical exponents in Equilibrium Statistical Mechanics using the Monte Carlo method as Metropolis [3], Swendsen-Wang [4], Wang-Landau [5] algorithms, Single histogram [6] and Broad histogram [7] methods. The Ising model was already applied

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decades ago to explain how a school of fish aligns into one direction for swimming [8] or how workers decide whether or not to go on strike [9]. In the Latané model of Social Impact [10] the Ising model has been used to give a consensus, a fragmentation into many different opinions, or a leadership effect when a few people change the opinion of lots of others. To some extent the voter model of Liggett [11] is an Ising-type model: opinions follow the majority of the neighbourhood, similar to Schelling [12], all these cited model and others can be found in [13]. Recently, Zaklan et al. [14, 15] developed an economics model to study the problem of tax evasion dynamics using the Ising model through Monte-Carlo simulations with the Glauber and heatbath algorithms (that obey detailed balance-equilibrium) to study the proposed model.

The beauty and the popularity of this model lies in both its simplicity and possible applications from pure and applied physics, via life sciences to social sciences. In the way similar to the percolation phenomenon, the Ising model is one of the most convenient way of numerical investigations of second order phase transitions.

In the simplest case, the Ising model may be used to simulate the system of interacting spins which are placed at the nodes of graphs or regular lattices. In its basic version only two values of the spin variable are available, i.e., $S=-1/2$ and $S=+1/2$. This is the classical Ising $S=1/2$ model. For a square lattice this model defines the universality class of phase transitions with analytically known critical exponents which describe the system behaviour near the critical point. The critical point separates two-ordered and disordered-phases.

One of possible generalization of the Ising model is to enlarge the set of possible spin values (like in the Potts model [16, 17]). The Ising $S=1$ model corresponds to three possible spin values, i.e., $S\in\{-1,0,+1\}$, Ising $S=3/2$ allows for four spin variables $S\in\{\pm3/2,\pm1/2\}$, etc. The Ising $S\neq 1/2$ model on various networks and lattices may form universality classes other than the classical square lattice Ising model.

The spin models for $S=1$ were extensively studied by several approximate techniques in two and three dimensions and their phase diagrams are well known [18–24]. The case $S>1$ has also been investigated according to several procedures [25–31]. The Ising model $S=1$ on directed Barabási-Albert network was studied by Lima in 2006 [32]. It was shown, that the system exhibits a first-order phase transition. The result is qualitatively different from the results for this model on a square lattice, where a second-order phase transition is observed.

The Archimedean lattices are the vertex transitive graphs that can be embedded in the plane such that every face is a regular polygon. A polygon is regular if all edges have the same length and all interior angles are the same. Kepler [33] showed that there exist exactly 11 such graphs. The lattices are given names according to the sizes (number of sides of the polygon) of faces incident to a given vertex. The face sizes are listed in order, starting with a face such that the list is the smallest possible in lexicographical order. The square lattice thus gets the name (4, 4, 4, 4), abbreviated to (4^4), triangular (3^6), honeycomb (6^3) and the Kagomé lattice the name (3, 6, 3, 6).

In this paper we study the Ising $S=1$ model on two Archimedean lattices (AL),