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A Level Set Method for the Inverse Problem of Wave Equation in the Fluid-Saturated Porous Media

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Abstract. In this paper, a level set method is applied to the inverse problem of 2-D wave equation in the fluid-saturated media. We only consider the situation that the parameter to be recovered takes two different values, which leads to a shape reconstruction problem. A level set function is used to present the discontinuous parameter, and a regularization functional is applied to the level set function for the ill-posed problem. Then the resulting inverse problem with respect to the level set function is solved by using the damped Gauss-Newton method. Numerical experiments show that the method can recover parameter with complicated geometry and the noise in the observation data.

AMS subject classifications: 35R30, 34K28

Key words: Level set method, fluid-saturated porous media, parameter recovery, damped Gauss-Newton method.

1 Introduction

The wave propagation theory in the fluid-saturated porous media was first established by Biot [1, 2], in which equations for acoustic propagation in a porous elastic isotropic solid containing a viscous fluid have been developed. Over the past decades, Biot theory has become a popular model for presenting the property of elastic wave propagation in the fluid-saturated porous media. In this paper, we consider the inverse problem with the coupling governing equations of **u-w** form given by Biot [3]:

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\mathrm{grad}(\mathrm{div}\mathbf{u})+\alpha M\mathrm{grad}(\mathrm{div}\mathbf{w})=\rho\frac{\partial^2\mathbf{u}}{\partial t^2}+\rho_f\frac{\partial^2\mathbf{w}}{\partial t^2}, \tag{1.1a}$$

$$\alpha M\mathrm{grad}(\mathrm{div}\mathbf{u})+M\mathrm{grad}(\mathrm{div}\mathbf{w})=\rho_f\frac{\partial^2\mathbf{u}}{\partial t^2}+m\frac{\partial^2\mathbf{w}}{\partial t^2}+\frac{\eta}{k}\frac{\partial\mathbf{w}}{\partial t}, \tag{1.1b}$$

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where **u** denotes the solid-frame displacement, **w** is the fluid displacement relative to solid-frame, λ is the Lamé coefficient, the quantity κ is the Darcy permeability coefficient, η is the viscosity of the pore fluid, ρ_f is the density of the pore fluid, ρ_s is the density of the solid grain, ρ is the bulk density defined by $\rho=\beta\rho_f+(1-\beta)\rho_s$, and β is the porosity. The relationship of the tortuosity α, added mass density m, and coupling constant M is given by

$$\alpha=1-\frac{K_s}{K_r}, \quad M=\frac{K_r^2}{D_r-K_s}, \quad D_r=K_r[1+\beta(\frac{K_r}{K_f}-1)], \quad m=\frac{\rho_f}{\beta},$$

where K_r is the bulk modulus of the grain, K_f is the bulk modulus of the pore fluid, and K_s is the bulk modulus of the skeletal frame.

The inverse problem discussed in this paper is to identify the porosity β from the measurements of the solid-frame displacement **u**, which can be viewed as a parametric data-fitting problem. It is possible to formulate such problem as an optimization problem where a functional defined by the discrepancy between the observed and computed data is minimized over a model space. In general, such problem is very difficult to solve, since it is nonlinear and ill-posed. For approximating the ill-posed and nonlinear problem, we utilize a regularized level set method. Level set method, originally introduced by Osher and Sethian [11] is a general framework for computation of evolving interfaces using the implicit representations and has been used successfully in many fields such as image processing [9, 10]. Hintermuller and Ring [7] applied a level set approach for the image segmentation. Recently, the level set method has received growing attention as a flexible algorithm for inverse problem [12] and shape optimization due to the ability to handle topological changes and to compute reconstructions with the minor priori information. Burger [4] studied the level set solution of shape reconstruction problems, in which the rigorous mathematical theory of level set regularization was established. van den Doel and Ascher [6] considered the level set regularization to recover the distributed parameter function with discontinuities for highly ill-posed inverse problem and the regularization functional was applied to the level set function rather than to the discontinuous function to be recovered. Tai and Li [13] applied a piecewise constant level set method to elliptic inverse problems and used the variational penalization method with the variation regularization of the coefficients to solve the inverse problem. Rondi [14] considered the regularization of the inverse conductivity problem with discontinuous conductivities.

In this paper, we will apply a level set method for parameter identification problem with the wave equations in the fluid-saturated porous media. For convenience, we only treat the case of the solution domain $\Omega=\Omega_1\cup\Gamma\cup\Omega_2$, in which the materials in two subregions Ω_1 and Ω_2 are different and separated by closed interface Γ. We assume the unknown parameter β takes the value β_1 in Ω_1 and takes the value β_2 in Ω_2. The purpose of this paper is that instead of recovering the parameter β, we want to implicitly capture the interface Γ by solving the optimal problem for the level set function, as Γ is usually the zero isocontour of the level set function. First, the unknown parameter β can be presented by a level set function. Then we formulate the inverse problem by minimiz-